Induction for Whole Numbers

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Professional Development from Texas Instruments

Student Activity

7 8 9 10 **11 12**

TI-84PlusCE™ | Investigation | Student | 30 min

Introduction

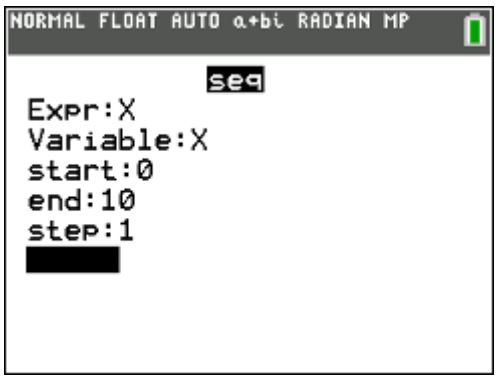
The purpose of this activity is to use exploration and observation to establish a rule for the sum of the first n whole numbers then use proof by induction to show that the rule is true for all whole numbers.

Calculator Instructions

The sequence command can be used to generate the first 10 whole numbers.

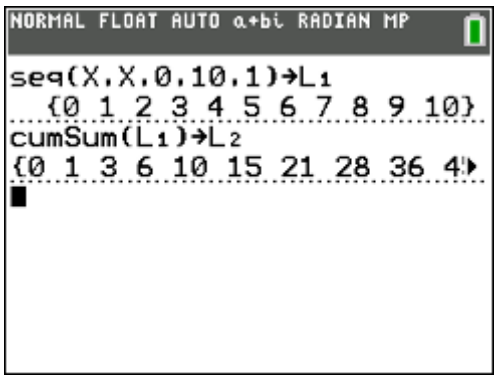
These values could be entered directly into a list, however it is often handy to know where and how to use some of the calculator's commands so that when longer lists need to be generated they don't have to be entered individually.

[2nd] [STAT] List > OPS > SEQ



Populate the sequence template as show opposite and paste into the calculator's home screen.

Store the generated list into L_1.



The cumulative sum of these numbers can also be computed.

[2nd] [STAT] List > OPS > CumSum

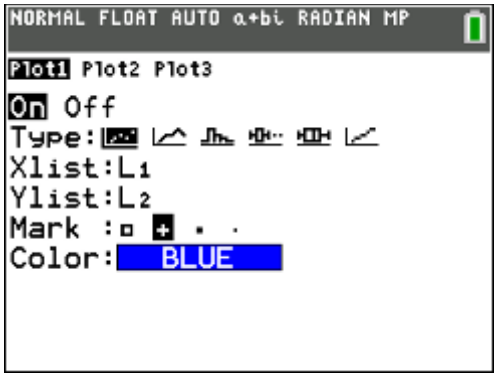
Store this cumulative sum in L_2.

Set up a scatter plot to graph the points where L_1 is plotted on the x axis and L_2 on the y axis.

[2nd] [Y=] Stat Plot > Plot 1

Make sure no other graphs are being plotted; then zoom in on the data.

Zoom > ZoomStat



Question 1

With reference to a difference table, explain why the relationship must be quadratic.

Authors: Peter Fox

Question 2

Use simultaneous equations to establish values for a, b and c where the sum (s) can be expressed in the form:

$$s = ax^2 + bx + c\,.$$

Question 3

Graph your equation to check that it passes through the points that have been plotted. Use $Y_1(x)$ and substitute a range of values to check your answer.

Question 4

Use induction (outlined below) to prove that your rule is true for all positive whole numbers.

a) Show that your rule is true for $x = 1$

b) Assume the result for $x = n$ is true and show that the rule holds for $n + 1$

Authors: Peter Fox