

Math Objectives

- Generalize a variety of patterns involving side lengths and angle measurements to draw conclusions about the properties of a parallelogram.
- Determine additional properties of rectangles, rhombi, and squares.

Vocabulary

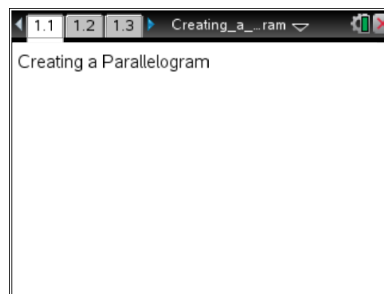
- parallelogram
- diagonal
- consecutive
- rhombus
- rectangle
- square

About the Lesson

- In this activity, students construct a parallelogram or use a premade construction.
- Students begin by dragging vertices of the constructed parallelogram to visualize the difference between constructed and drawn figures.
- Students then use measurement tools to generalize a variety of patterns involving side lengths and angle measurements.
- As a result, students will:
 - Manipulate and visualize the parallelogram as they are dragging vertices.
 - Draw conclusions about the properties of a parallelogram.

TI-Nspire™ Navigator™ System

- Use Class Capture to monitor progress.
- Use Quick Poll to assess progress.



TI-Nspire™ Technology Skills:

- Open a document
- Move from one page to another
- Measure lengths
- Measure angles

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Files:

Student Activity

- Creating_a_Parallelogram_Student.pdf
- Creating_a_Parallelogram_Student.doc

TI-Nspire documents

- Creating_a_Parallelogram.tns
- Quads.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Teacher Preparation and Notes

- For information regarding the creation of the file *Creating_a_Parallelogram.tns*, refer to *Creating_a_Parallelogram_Create.doc*.
- Students can either create the *.tns* file by following the instructions provided in *Creating_a_Parallelogram_Create.doc* or they can use the pre-constructed file entitled *Creating_a_Parallelogram.tns*.
- This activity is intended to be introduced by the teacher and completed by the students as they discover and justify their conjectures.
- The student worksheet helps guide students through the activity and provides a place for students to record their answers.

There are various definitions given in textbooks for some of these special quadrilaterals. In this activity, the following definitions are used:

A **parallelogram** is a quadrilateral with opposite sides parallel.

A **rectangle** is a parallelogram with 4 right angles.

A **rhombus** is a parallelogram with all sides congruent.

A **square** is a parallelogram with 4 right angles and all sides congruent.

A **kite** is a quadrilateral with 2 pairs of adjacent sides congruent.

A **trapezoid** is a quadrilateral with 1 pair of opposite sides parallel.

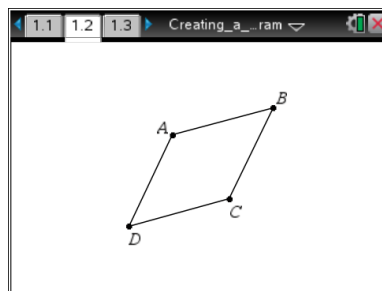
There are ongoing discussions about some definitions, particularly *kite* and *trapezoid*. Is a rhombus a kite? Is a parallelogram a trapezoid? Some sources say they are and others say they are not. This might be a good discussion to have with students. For this activity, a rhombus is a kite, but a parallelogram is not a trapezoid. If your source is different, adjust the activity accordingly.

Move to page 1.2 to begin the lesson.

Note: Pages 1.3 and 1.4 are identical to page 1.2. Use these pages as needed.

1. Manipulate the parallelogram by dragging the vertices.
(**Note:** if you did not construct the parallelogram, discuss a and b with your teacher)
 - a. Why is parallelogram $ABCD$ considered “constructed” and not just drawn? Explain.

Sample Answer: When vertices A , B , or C are dragged, the figure remains a parallelogram because construction tools were used to make the parallelogram.





Teacher Tip: If you did not have your students construct the parallelogram, explain the difference between constructing a figure and just drawing it. Constructed figures retain their properties when pulled and stretched, drawn figures do not.

- b. Can every vertex of the constructed parallelogram be dragged? Explain why or why not.

Sample Answer: Only vertices A , B , and C are independent points and available to be dragged. Vertex D is dependent on the construction.

Press **Menu > Measurement > Length** to measure the sides. Use the Touchpad to move the cursor to one of the sides and **tab** to select a side (not the perimeter) press **↩** to select the side. A “ghost” measurement will appear. Use the Touchpad to move the measurement to a desired location and press **↩** or **enter** again. Repeat for all sides.

TI-Nspire Navigator Opportunity: Live Presenter

See Note 1 at the end of this lesson.

2. Press **esc**. Drag a vertex of the parallelogram and describe the side-length relationships in the parallelogram.

Answer: Opposite sides are congruent.

Teacher Tip: If a measurement displays more decimal digits than desired, exit all tools, and move the cursor over the value. Press **+** to display more digits or **-** to remove digits.



To label a measurement, double enter on the measurement, arrow to the front of the measurement, and type. For example: $AB=$, and so on. (For capital letters, press the shift key before entering the letter.)

To move a measurement, drag as before.

Press **Menu > Measurement > Angle** to measure each of the interior angles. Angle measurements require selecting 3 points with the vertex of the angle selected second. Move the pencil cursor to the first point of the angle, and press **↩** or **enter** to select the first point. Move the cursor to the second point (vertex of the angle) and press **↩** or **enter**. Repeat for the third point. An angle arc and measurement will appear. Repeat for all interior angles.



Teacher Tip: If using the premade file, angles are set for measuring in degrees. If the measurement is in radians, use the document settings to change to degrees.



Teacher Tip: Because of rounding or dragging issues, angles may not appear to sum to 180° , since a fixed number of digits is displayed. To confirm that consecutive angles are supplementary, you may want to have students use the **Text** and **Calculate** tools. Press **Menu > Actions > Text**. Press and type the formula **a+b** in the text box. Press to finish the text. Press **Menu > Actions > Calculate**. Press or  on the formula and then  on each angle value.

TI-Nspire Navigator Opportunity: Quick Poll / Open Response
See Note 2 at the end of this lesson.

3. Press . Drag a vertex of the parallelogram and describe at least 2 angle relationships.

Sample Answer: Opposite angles are congruent. Consecutive angles are supplementary. All interior angles sum to 360° .

Move to page 1.4.

Press **Menu > Points & Lines > Segment**. Draw the diagonals of the parallelogram by moving the cursor to a vertex and pressing  or and moving to the opposite vertex and pressing  or . Repeat to draw the other diagonal.

4. Use measurement and construction tools to explore the diagonals. Measure the lengths of the diagonals. Find the point of intersection of the two diagonals and use that to find the lengths of the parts of the diagonals that are created when the diagonals intersect. When a vertex is moved, what can you conjecture about the diagonals of a parallelogram?

Sample Answer: Diagonals of a parallelogram bisect each other.

Teacher Tip: Encourage students to find the intersection of the two diagonals, measure each part, and so on.

Teacher Tip: Although these quadrilaterals are not in a coordinate plane, you may still want to have a discussion with students about the slopes of parallel lines through the sides of the parallelogram.



5. Rectangles, rhombi, and squares are special parallelograms. All properties of parallelograms hold true for these figures as well, but are there more? Drag vertices and measure other segments and angles as needed.
- a. Return to page 1.3 and drag the vertices until you get a rectangle. Are there any other special properties for a rectangle?

Answer: Diagonals of a rectangle are congruent.

- b. Return to page 1.2 and drag the vertices until you get a rhombus. What other special properties hold for a rhombus?

Answer: Diagonals of a rhombus are always perpendicular. Diagonals of a rhombus always bisect the angles of the rhombus.




- c. Return to pages 1.2 or 1.3 and drag the vertices until you get a square. What other special properties hold for a square?



Answer: Squares have all of the properties of parallelograms, rectangles, and rhombi. The triangles formed by constructing the diagonals of a square are 4 congruent right isosceles triangles. (See next investigation.)

Teacher Tip: It could be good practice to have students add pages to the document and **construct** the figures; rectangle, square, and rhombus. Then investigate their properties.

Teacher Tip: Sides or angles may be enough to find some of the special parallelograms. Encourage students to use other measurements (e.g., area) to justify their findings.

Return to page 1.4 to explore areas.

6. Press **Menu > Shapes > Triangle**. To create triangle ACD :
- a. Move the cursor to point A and press  or **enter**.
- b. Move the cursor to point C and press  or **enter**.
- c. Finally move the cursor to point D and press  or **enter**.
- d. Repeat to create $\triangle ACB$, $\triangle ADB$, and $\triangle DCB$. Press **esc** to exit the tool.

7. Press **Menu > Measurement > Area**.
 - a. Move the cursor to $\triangle ACD$ and press  or **enter** to select it. Move the cursor to a nearby location and press  or **enter** again to place the area measurement in the desired location. Repeat the process for the other 3 triangles.
 - b. Drag any of the vertices.
8. What is true about the areas of the triangles formed by the diagonals?

Answer: The areas are equal.

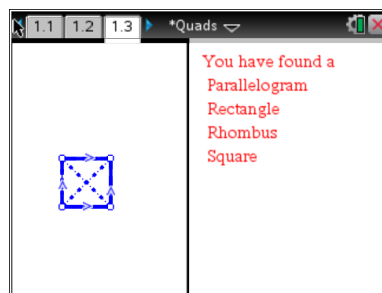
9.
 - a. Repeat step 6 to overlay triangles over each of the small triangles formed. When moving to the point where the diagonals intersect, it will say only *intersection point*.
 - b. Repeat step 7 to measure the areas of all of the small triangles.
10. Drag any of the vertices. What is true about the areas of the smaller triangles formed by both diagonals? What is special about the triangles?

Answer: The areas are equal. The triangles are isosceles.

TI-Nspire Navigator Opportunity: *Class Capture*
See Note 3 at the end of this lesson.

Extension – Exploring Another Quadrilateral

If desired, have students use the TI-Nspire™ document *Quads.tns*. A suggestion might be to have students come to the board and try the following with the file in the emulator view. It can be done on each handheld, but because of the complexity of the file, it will run slower on the handheld. This file can be used as a class assessment and will help review all of the properties of quadrilaterals. In place of using the file, you might ask students to summarize their findings about properties of special quadrilaterals.



Teacher Tip: Remind students to use **ctrl** **tab** to move between parts of a page.

On page 1.3, on the left side of the screen, have students drag any vertex to find different quadrilaterals. Encourage them to drag the vertices to get each one of the quadrilaterals (parallelogram, rectangle, square, kite, trapezoid). If necessary, students can use the **Measurement** tools to help them. Point out to students the different symbols to show parallelism, equal segments, and so on.

Ask: Why are all squares also rectangles?

Answer: By definition, a square is a rectangle because it has 4 right angles, but in addition, a square has all sides congruent.

Move to page 2.2

Have students drag the vertices on the left side of the page. Once they get a particular quadrilateral, drag the slider to what they think is the correct answer. Encourage students to make all of the different quadrilaterals by using the question slider.

Move to the other pages and have students answer the questions.

TI-Nspire Navigator

Note 1

Question 1, *Live Presenter*: Select one or two students to illustrate how to answer different parts of question 1.

Note 2

Question 3, *Quick Poll / Open Response*: Have students submit their answers for each question below.

In parallelogram ABCD:

1. The measure of angle A is 60 degrees. What is the measure of angle C?
2. The measure of angle A is 60 degrees. What is the measure of angle B?
3. The length of side AB is 6 cm. What other side has a length of 6 cm?

Answers: 1. 60 (degrees) 2. 120 (degrees) 3. Side CD

Note 3

Questions 6-10, *Class Capture*: While students are investigating questions 6–10, continually take Class Captures to see if they understand the questions asked.