

About the Lesson

In this activity, students will explore the behavior of various functions by zooming in on their graphs. They will investigate the concept of local linearity. As a result, students will:

- Make connections to many calculus concepts including slope, limit of a difference quotient, the definition of the derivative, and the criteria for differentiability.
- Identify slope of the tangent.
- Make connections to definition of the derivative and differentiability.

Vocabulary

- slope
- tangent line
- cusp

Teacher Preparation and Notes

- Students will receive significant exposure to piecewise functions in this activity and will use the calculator to graph them.
- The future topics of limits and derivatives at a point that will be covered in class can relate back to this activity quite adequately.

Activity Materials

- Compatible TI Technologies:

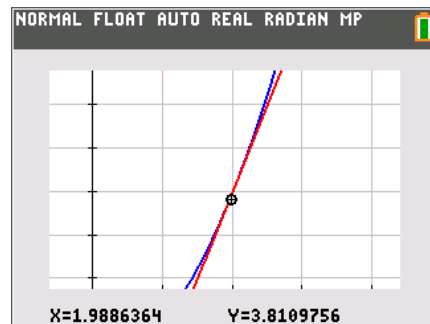
TI-84 Plus*

TI-84 Plus Silver Edition*

 TI-84 Plus C Silver Edition

 TI-84 Plus CE

* with the latest operating system (2.55MP) featuring MathPrint™ functionality.



Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Zooming_In_On_Local_Linearity_Student.pdf
- Zooming_In_On_Local_Linearity_Student.doc



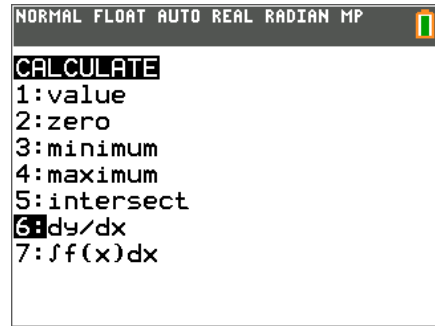
Problem 1 – Draw a Tangent Line by Hand

Students will begin the activity by drawing a line tangent to the graph of $y = x^2$ at some point in the first quadrant. Students can approximate the slope and equation of the tangent line using the grid points on the graph.

Problem 2 – Draw and Explore Tangent Lines Using Technology

Students will graph the function $Y_1=X^2$ and draw a tangent line as they did in Problem 1.

Students will then be instructed to calculate the slope of the tangent at the value of x they previously chose by pressing $\boxed{2nd} \boxed{trace} \boxed{[calc]}$ and selecting option **6:dy/dx**.



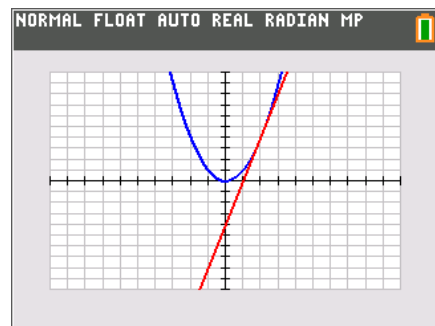
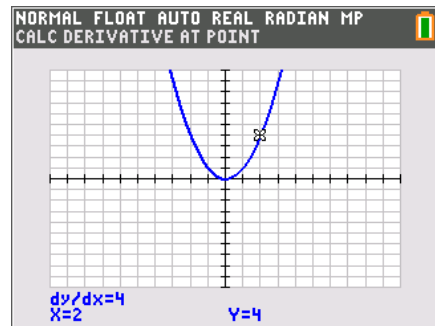
Next, type the value of x where that indicates the position of the point of tangency.

Note: For the purpose of this demonstration, $x = 2$ is the location of the point of tangency.

From here students will be required to write the equation of their tangent line using the point-slope form of a line, $y - y_1 = m(x - x_1)$.

Once they solve this equation for y , it may be entered into the $Y_2=$ entry line.

In this case, $Y_2= 4x - 4$. Pressing \boxed{graph} will show both the curve and the tangent line.

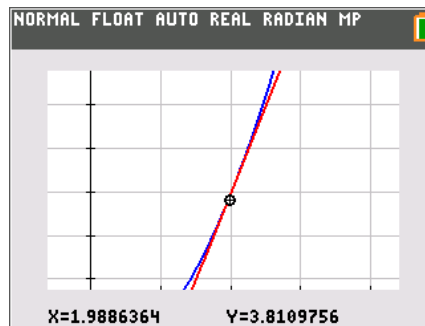




Next, students will be instructed to zoom in and observe the behavior of both the curve and the tangent line.

Press `[zoom]` and select the option **2:Zoom In**.

Move the cursor near the point of tangency chosen from Problem 1 and press `[enter]`.



Students can repeat this process by simply pressing `[enter]`. They may find it helpful to reposition the cursor over the desired point of tangency. It shouldn't bother students if they cannot position the cursor exactly on the point of tangency. Encourage them to get as close as possible.

Repeat the process of zooming in on the point of tangency a couple more times to observe what happens.

1. Write your observation of how your tangent line and the graph $Y_1=X^2$ compare when examined close up.

Answer: When examined up close, the tangent line and the graph of $y = x^2$ seem to be the same line.

2. Will this type of behavior occur for all other functions? Explain your reasoning.

Sample Answer: No, because some functions are not continuous everywhere.

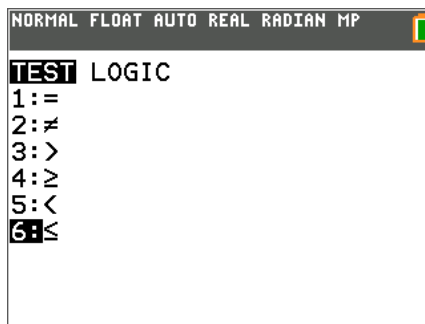
Problem 3 – Graph a Piecewise Function to Explore Local Linearity

In Problem 3, students will graph the piecewise function

$$y = \begin{cases} x^2, & x \leq 2 \\ 2x, & x > 2 \end{cases}$$

and observe whether or not the graph is

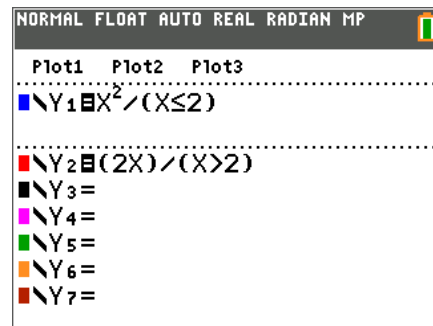
locally linear in the neighborhood of (2, 4).





To graph a piecewise function, students will have to use both the $Y_1=$ and $Y_2=$ entry lines. The two parts of the piecewise function must be entered as shown in the screenshot below and to the right.

To type an inequality, press 2nd math [test] and select the desired symbol.



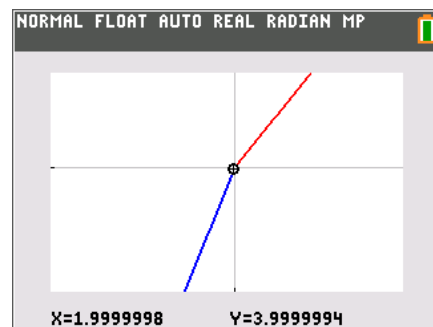
Students will observe that some piecewise functions have the local linearity property, while other piecewise functions lack this property.

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3. What is happening in the neighborhood of (2, 4) for the piecewise function $y = \begin{cases} x^2, & x \leq 2 \\ 2x, & x > 2 \end{cases}$?

Does this function demonstrate local linearity at that location?

Answer: No, it does not demonstrate one straight line. It appears to be a sharp turn.



Discuss with students that this type of point is referred to as a “cusp.”

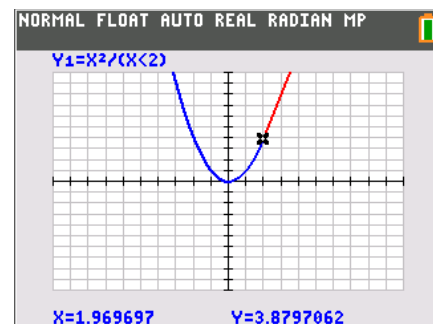
Problem 4 – Graphing Another Piecewise Function

Students will graph the piecewise function

$$f(x) = \begin{cases} x^2, & x < 2 \\ 4x - 4, & x \geq 2 \end{cases}$$

using the same procedure from Problem

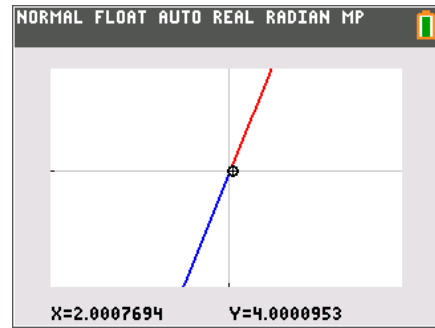
3. Again, students should begin by viewing the graph with **Zoom Standard** and then **Zoom In** a few times.





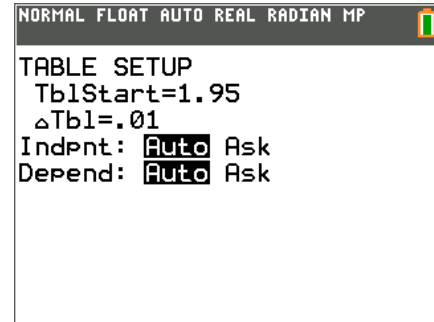
4. Does this function appear to be locally linear in the neighborhood of (2, 4)? Compare and contrast this function to the one graphed and explored in Problem 3.

Answer: Yes, both parts of the piecewise function seem to meet linearly at the point (2, 4).



Students can find additional evidence for their findings by viewing a table of values for their function.

First they will have to set up their table appropriately by pressing $\boxed{2nd} \boxed{window} \boxed{[tblset]}$ and changing **Tblstart**=1.95 and **ΔTbl** =0.1.



Students should take note of the relative closeness of the y-values as the x-values are very close to 2.

X	Y1	Y2
1.95	3.8025	ERROR
1.96	3.8416	ERROR
1.97	3.8809	ERROR
1.98	3.9204	ERROR
1.99	3.9601	ERROR
2	ERROR	4
2.01	ERROR	4.04
2.02	ERROR	4.08
2.03	ERROR	4.12
2.04	ERROR	4.16
2.05	ERROR	4.2

Problem 5 – Conclusion

5. If you were finding the slope of function in the interval of a repeatedly zoomed in graph, describe what happens to $\Delta x = x_2 - x_1$.

Answer: As the interval of a graph is repeatedly zoomed in, Δx becomes closer to zero.