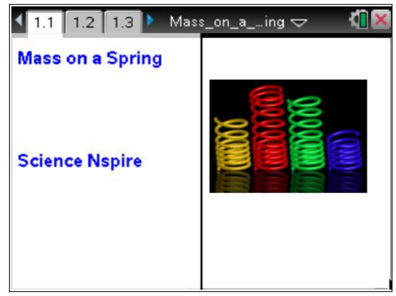
Science Objectives

- Students will investigate the effects of mass, spring constant, and initial displacement on the motion of an object oscillating on a spring.
- Students will observe the relationships between displacement, velocity, and acceleration for an object in simple harmonic motion.



Vocabulary

- amplitude
- period
- simple harmonic motion (SHM)
- spring constant

About the Lesson

- This lesson simulates the motion of an anvil attached to a spring as it oscillates on a frictionless, horizontal surface. Students are able to manipulate the mass of the anvil, the spring constant of the spring, and the initial displacement of the anvil from equilibrium. The motion is analyzed by using position vs. time, velocity vs. time, and acceleration vs. time graphs.
- As a result, students will:
  - Describe the effects of mass, spring constant, and initial displacement on the motion.
  - Identify the factors that affect the period.
  - Calculate the period, maximum speed, and maximum acceleration.

TI-Nspire™ Navigator™

- Send out the *Mass_on_a_Spring_Student.tns* file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes class captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Mass_on_a_Spring_Student.doc
- Mass_on_a_Spring_Student.pdf

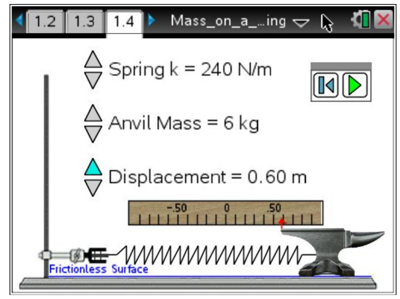
TI-Nspire document

- Mass_on_a_Spring.tns

Discussion Points and Possible Answers

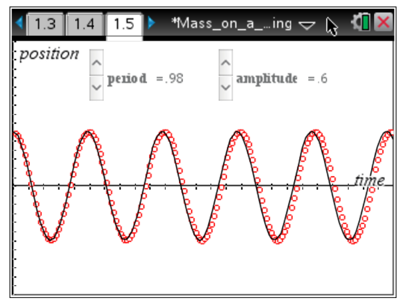
Move to pages 1.2 – 1.6.

Have students read the information on pages 1.2 and 1.3.



1. When the anvil on page 1.4 is at a position of $x = 0$, the spring is neither stretched nor compressed; at this position the spring applies no force to the anvil. When the anvil is moved from this position, the spring applies a force to the anvil back toward $x = 0$. The magnitude of this force is equal to the spring constant of the spring, k, multiplied by the displacement of the anvil from $x = 0$.

   In the simulation, students set the anvil's mass ($2\text{ kg} \le m \le 10\text{ kg}$), the spring constant of the spring ($40\text{ N/m} \le k \le 300\text{ N/m}$), and the initial position of the anvil ($-1 \le x_i \le 1$ m).



   Selecting the Play button on the screen causes the anvil to oscillate around $x = 0$. Selecting the Pause button will stop the motion. A position vs. time graph of the anvil's motion is plotted on page 1.5 following the simulation. The Reset button clears the graph and resets the simulation. Following the position vs. time graph page are velocity vs. time and acceleration vs. time graphs of the anvil's motion on page 1.6.

Tech Tip: To access the Directions again, select menu or **Document Tools ( ) > Mass on a Spring > Directions.**

Tech Tip: To access the Directions again, select > **Directions.**

Refer back to the simulation on pages 1.4. and 1.5

Have students answer the questions on their activity sheets.

Q1. Choose mid-range values for the mass of the anvil, the spring constant, and the initial position of the anvil. Record your values below. Select the Play button ▶ and observe the motion of the anvil as it moves through several cycles of its motion.

Suggested Answer: The mass can vary between 2 kg and 10 kg. The spring constant can vary between 40 N/m and 300 N/m, and the initial position can vary between -1.0 m and 1.0 m. Later in the activity students will increase the value of each variable. Encourage them to start with values in the lower mid-range of each variable.

> **Teaching Tip:** Instruct students to move to page 1.5 while the anvil is oscillating to watch as the position vs. time graph of the motion is plotted.

Q2. View the position vs. time graph of the anvil's motion. Describe the motion of the anvil based on the data points you see (the open circles).

Suggested Answer: The mass oscillates back and forth on either side of $x = 0$. The motion repeats after a fixed amount of time. The curve looks like a cosine curve because it starts at a positive value.

Q3. Simple Harmonic Motion is characterized by period and amplitude. The period of the motion, T, is the time to complete one full cycle. The amplitude of the motion, A, is the maximum displacement from equilibrium ($x = 0$). Use the sliders at the top of page 1.5 to adjust the period and amplitude of the plotted function (the solid curve) so that it fits the data points. Record the values below.

Suggested Answer: Answers will vary based on the values of the variables the students choose. The amplitude should match the value for initial position they chose in question 1. The period of the function should be adjusted so that the function fits all of the cycles of the data well and not just the first cycle.

TI-Nspire Navigator Opportunities

Make a student a Live Presenter to demonstrate how to determine amplitude and period.

Q4. Reset the simulation on page 1.4. Increase the mass of the anvil. Keep the spring constant and the initial position set to the values you recorded in question 1 above. Describe how this changes the motion. Record the mass, period, and amplitude below.

Suggested Answer: Specific values will vary. Encourage students to increase the mass significantly. They should observe that the period of the motion increases when the mass increases, but the amplitude of the motion does not change. Students should conclude that the anvil must move slower than before.

Q5. Reset the simulation. Increase the spring constant, but keep the mass and initial position set to the values you recorded in question 1 above. Describe how this changes the motion. Record the spring constant, period, and amplitude below.

Suggested Answer: Specific values will vary. Encourage students to increase the spring constant significantly. They should observe that the period of the motion decreases when the spring constant increases, but the amplitude of the motion does not change. Students should conclude that the anvil must move faster than before.

Q6. Reset the simulation. Increase the initial position, but keep the mass and spring constant set to the values you recorded in question 1 above. Describe how this changes the motion. Record the initial position, period, and amplitude below.

Suggested Answer: Specific values will vary. Encourage students to increase the initial position significantly. They should observe that the period of the motion does not change, but the amplitude of the motion increases to match the new initial position. Students should conclude that the anvil must move faster than before.

Q7. Reset the simulation, and set the mass, the spring constant, and the initial position to the values you used in question 1. Run the simulation once again. Consider the position vs. time graph on page 1.5. Use the **Graph Trace** tool (**Menu > Trace > Graph Trace**) to determine two times near the beginning of the motion when the anvil passes through x = 0 and two times when the anvil is at maximum displacement. Record these four measurements below. What function would produce this graph?

Suggested Answer: The recorded times will vary from student to student. Some students may not be familiar with the graph of the cosine function, but others may recognize it. The actual function is

$$x = A \cdot \cos\left(\sqrt{\frac{k}{m}} \cdot t\right) = A \cdot \cos\left(\frac{2p}{T} \cdot t\right).$$

Tech Tip: To use the **Graph Trace** tool, have students select > **Trace > Graph Trace.** Then, students can drag their finger along the graph to move the cursor.

Q8. According to one of the equations of SHM, the period of the motion is calculated by the equation $T = 2p\sqrt{\frac{m}{k}}$. Substitute your values for *m* and *k* into this equation, and calculate the period of the anvil's motion. Show your work below, and compare this to the period you recorded in question 3.

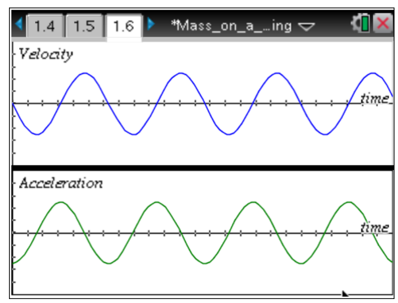
Suggested Answer: Answers depend upon the values students choose.

TI-Nspire Navigator Opportunities

Make a student a Live Presenter to demonstrate how to calculate the period.

Refer back to the simulation on page 1.6.

Have students answer the questions on the activity sheet.



2. This page displays the velocity vs. time graph of the anvil's motion on top and the acceleration vs. time graph on bottom. Students may need to adjust the scales on the *y*-axes to see the graphs well. Students should select **Menu or** **> Window / Zoom > Window Settings**. They will adjust the **YMin** and **YMax** values to make the graphs fit the windows. Students will need to do this for each window separately.

Q9. How are these two graphs similar to the position vs. time graph on the previous page?

Suggested Answer: Both graphs are periodic and have the same period as the position vs. time graph. Both graphs appear to be graphs of trig functions.

Q10. How is the shape of the velocity vs. time graph different from the position vs. time graph? What function could produce this graph?

Suggested Answer: The graph starts from the origin and goes down initially. It is like a sine function except it goes down initially instead of up. It could be the sine function multiplied by a negative coefficient. The actual function is $v = -A \cdot \sqrt{\frac{k}{m}} \cdot \sin\left(\sqrt{\frac{k}{m}} \cdot t\right)$.

Q11. Use the **Graph Trace** tool to determine two times near the beginning of the motion when the velocity of the anvil is zero and two times when the velocity is at its maximum value. Record these measurements below. Compare these times to the times you recorded in question 7 above. What can you infer about the relationship between the velocity of the anvil and its position?

Suggested Answer: The velocity is zero when the position is at its maximum, and the velocity is maximum when the position is zero. They are out of sync by a quarter of a cycle.

Q12. Use your values of *m*, *k*, and *A* to calculate the expression $A \times \sqrt{\frac{k}{m}}$. Show your work below. Use the **Graph Trace** tool to determine the maximum speed of the anvil. How do these two values compare?

Suggested Answer: Specific answers will vary, but students should find that the calculation matches the maximum speed of the anvil from the graph.

Q13. How is the shape of the acceleration vs. time graph different from the position vs. time graph? What function could produce this graph?

Suggested Answer: The acceleration starts as negative instead of positive. It is the same shape as the position graph flipped over. The actual function is $a = -A \cdot \frac{k}{m} \cdot \cos\left(\sqrt{\frac{k}{m}} \cdot t\right)$.

Q14. Use the **Graph Trace** tool to determine two times near the beginning when the acceleration of the anvil is zero and two times when the acceleration is at its maximum value. Record these measurements below. Compare these times to the ones you recorded in question 7 above. What can you infer about the relationship between the acceleration of the anvil and its position?

Suggested Answer: The acceleration is zero when the position is zero, and the acceleration is maximum when the position is maximum. The acceleration always has the opposite sign as the position.

Q15. Use your values of *m*, *k*, and *A* to calculate $A \times \frac{k}{m}$. Show your work below. Use the **Graph Trace** tool to determine the anvil's maximum acceleration. How do these two values compare?

Suggested Answer: Specific answers will vary, but students should find that the calculation matches the maximum acceleration of the anvil from the graph.

Q16. Use the equations given in questions 8, 12, and 15 to calculate the period, maximum speed, and maximum acceleration of the anvil when *m* = 8.0 kg, *k* = 220 N/m, and *A* = 0.60 m. Show your work below. Use the simulation to verify your results.

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad v_{max} = A \cdot \sqrt{\frac{k}{m}} \qquad a_{max} = A \cdot \frac{k}{m}$$

Suggested Answer:

$$T = 2\pi\sqrt{\frac{8.0}{220}} \qquad v_{max} = 0.60 \cdot \sqrt{\frac{220}{8.0}} \qquad a_{max} = 0.60 \cdot \frac{220}{8.0}$$

$$T = 1.20 \text{ s} \qquad v_{max} = 3.1 \text{ m/s} \qquad a_{max} = 16.5 \text{ m/s}^2$$

Move to pages 1.7 – 1.13.

Have students answer the questions on either the handheld, on the activity sheet, or both.

Q17. The period of motion of an object on a spring is determined by _______________. (More than one response may be correct.)

Answer: A. the mass of the object, and C. the spring constant of the spring

Q18. Increasing the mass will increase the period of the motion.

Answer: A. True

Q19. Increasing the spring constant will increase the period of the motion.

Answer: B. False

Q20. The velocity of the object _______________.

Answer: D. is maximum when displacement is zero

Q21. Select all of the factors below that will increase the maximum velocity of the object. (More than one response may be correct.)

Answer: B. increase the amplitude of the motion; C. increase the spring constant; and F. decrease the mass.

Q22. The magnitude of the acceleration _______________.

Answer: B. is maximum when the displacement is maximum

Q23. The direction of the acceleration is _______________.

Answer: A. opposite to the direction of displacement

TI-Nspire Navigator Opportunities

Throughout the activity, discuss the simulation and graphs with students using Slide Show. At the end of the lab, collect the .tns files and save to Portfolio.

Wrap Up

When students are finished with the activity, pull back the .tns file using TI-Nspire Navigator. Save grades to Portfolio. Discuss activity questions using Slide Show.

Assessment

- Formative assessment will consist of questions embedded in the .tns file and student responses on the handout. The questions will be graded when the .tns file is retrieved. The Slide Show will be utilized to give students immediate feedback on their assessment.
- Summative assessment will consist of questions/problems on the chapter test.