



Science Objectives

- Students will explore the relationship between a vector and its components.
- Students will explore how two vectors may be combined to produce a resultant vector.
- Students will be able to calculate the resultant of two vectors which are added.

Vocabulary

- | | |
|--------------------|---------------------------|
| • clockwise | • perpendicular |
| • collinear | • quadrant |
| • component | • resultant |
| • counterclockwise | • magnitude and direction |
| • parallel | |

About the Lesson

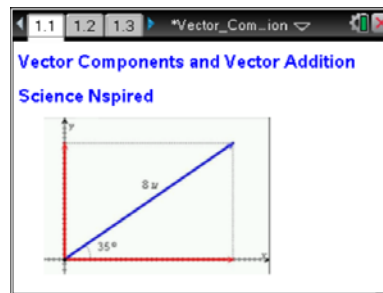
- This lesson first explores the relationship between the magnitude and direction of a vector and the components of the vector.
- This lesson then explores how two vectors are added to produce a resultant vector. Vector addition progresses from collinear vectors to perpendicular vectors and finally to general vectors.
- As a result, students will:
 - Describe how the magnitude and direction of a vector determine its components.
 - Calculate the magnitude and direction of the resultant vector when two perpendicular vectors are added.
 - Determine the resultant vector when two general vectors are added by using the components of the vectors.

TI-Nspire™ Navigator™

- Send out the *Vector_Components_and_Vector_Addition.tns* file.
- Monitor student progress using Screen Capture.
- Use Live Presenter to spotlight student answers.
- Collect students' .tns files and evaluate student understanding.

Activity Materials

- *Vector_Components_and_Vector_Addition.tns* document
- TI-Nspire™ Technology
- Student Handout



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Use a minimized slider

Tech Tips:

To change the value of a length or angle slider, students may either click on the arrows, or they may click the number and change it directly.

Lesson Materials:

Student Activity

- *Vector_Components_and_Vector_Addition_Student.doc*
- *Vector_Components_and_Vector_Addition_Student.pdf*

TI-Nspire document

- *Vector_Components_and_Vector_Addition.tns*



Discussion Points and Possible Answers

Move to pages 1.2–1.4.

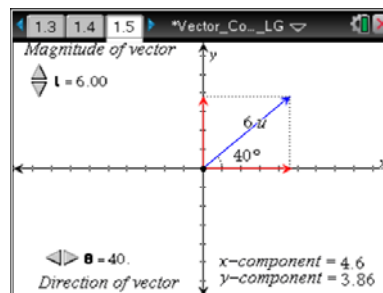
1. Students should read the introductory material on pages 1.2, 1.3, and 1.4.

Tech Tip: Remind students to change the value of a length or angle slider you may either click on the arrows, or you may click the number and change it directly.

Move to page 1.5.

Have students answer the questions on the activity sheet.

2. Students should manipulate the vector by adjusting the upper slider that controls the magnitude of the vector (0 to 10 units) and the lower slider that controls the direction of the vector (0° to 360°). The direction is measured counter-clockwise from the positive x-axis.



- Q1. With the vector in the first quadrant, change the length of the vector while keeping the angle constant. Describe what happens to the x and y components of the vector as you increase and decrease its length.

Sample Answer: As the lengths of the components increase, the length of the vector increases. As the length of the components decrease the length of the vector decreases.

- Q2. Double the length of the vector. How do the x and y components change as a result of this increase in length?

Sample Answer: The x and y components double in length also.

- Q3. Keep the length of the vector constant and manipulate the angle to move the vector within the 1st quadrant. What happens to the components as the vector moves toward the x-axis (decreasing the angle)?

Sample Answer: The x component increases, and the y component decreases.

- Q4. What happens to the components as the vector moves toward the y-axis (increasing the angle)?

Sample Answer: The x component decreases, and the y component increases.



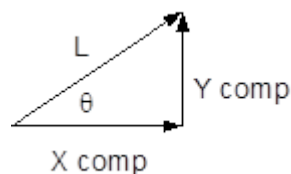
Q5. At what angle are the two components equal?

Answer: 45°

Q6. Set the length of the vector to 8 units and the direction to 30° . Write the values of the x and y components below.

Answer: x component = 6.93 y component = 4

Q7. The y component can be drawn from the tip of the x component so that the two components form the sides of a right triangle. The vector will be the hypotenuse. Using trigonometry and this picture, write equations for calculating the x and y components of a vector with length L and angle θ . Verify that your equations work by using them to calculate the components of the vector in the previous question.



Sample Answer: (Depending on the backgrounds of the students, the teacher may need to review basic trig relationships at this point.)

$$\text{x component} = L \cdot \cos\theta = 8 \cdot \cos 30^\circ = 6.93$$

$$\text{y component} = L \cdot \sin\theta = 8 \cdot \sin 30^\circ = 4$$

Q8. Increase the angle of the vector so that it is greater than 90° and is in the 2nd quadrant. As you move the vector into the 2nd quadrant, what happens to the sign and lengths x and y components?

Sample Answer: The x component becomes negative, but the y component remains positive. The length of the x component increases and the y component decreases.

Q9. Set the vector length to 8 units and the direction to 115° . Write the values of the x and y components below. Verify that your equations for calculating the components give you the same results.

Answer: x component = $8 \cdot \cos 115^\circ =$ -3.38 y component = $8 \cdot \sin 115^\circ =$ 7.25



Q10. What happens to the x and y components as you move the vector into the third and fourth quadrants?

Sample Answer: Both the x and y components are negative in the 3rd quadrant. The y component increases and the x component decreases as the vector moves into the 3rd quadrant. The x component is positive and the y component is negative in the 4th quadrant. The x component increases and the y component decreases as the vector moves into the 4th quadrant.

Vector Addition

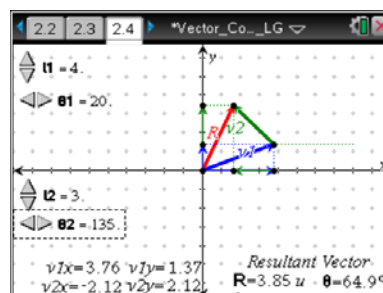
The remainder of the document is focused on vector addition. In problem 2, students explore the addition of collinear vectors. Problem 3 deals with perpendicular vectors. In problem 4, students explore using components to add two vectors with any orientation.

Move to pages 2.2 and 2.3.

3. Students should read the background material on pages 2.2 and 2.3 before moving to the simulation on page 2.4. On page 2.4, students should set the length of vector 1 to 5 units and the direction to 20° . Vector 2 should be set to a length of 3 units.

Move to page 2.4.

Have students answer the questions on the activity sheet.



Q11. Set vector 1 to a length of 5 units and a direction of 20° . Set vector 2 to a length of 3 units. Describe the resultant vector when you make the direction of vector 2 the same as vector 1 (20°).

Sample Answer: The resultant vector points in the same direction (20°) and lays on top of the other two. Its length is the sum of their lengths.

Tech Tip: To change the value of a length or angle slider, students may either click on the arrows, or they may click the number and change it directly.



Q12. Change the direction of vector 2 so that it points opposite to vector 1 ($\theta_2 = 200^\circ$). Describe the resultant vector.

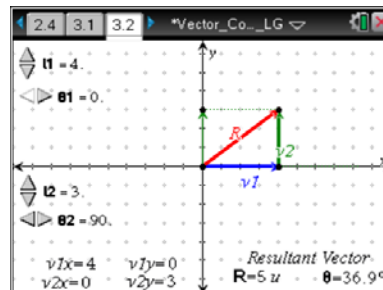
Sample Answer: The resultant vector points in the same direction (20°) as the longer vector. Its length is the difference between the two lengths.

Perpendicular Vectors

Some students may need a review of trigonometry. This section could be extended to provide students more practice.

Move to page 3.2.

Have students answer the questions on the activity sheet.



Q13. When two vectors are perpendicular, what equation relates the length of their resultant vector, R , to the lengths of the two vectors, l_1 and l_2 ? Show that this equation works when $l_1 = 4$ units and $l_2 = 3$ units.

Sample Answer: $R = \sqrt{l_1^2 + l_2^2}$ $R = \sqrt{4^2 + 3^2} = 5$ units

TI-Nspire Navigator Opportunities

Make a student a Live Presenter to illustrate the calculation of the magnitude and direction of the resultant vector.

Q14. What equation can be used to calculate the angle of the resultant vector using l_1 and l_2 (think about trigonometry)? Show that this equation works when $l_1 = 4$ units and $l_2 = 3$ units.

Sample Answer: $\theta = \tan^{-1}\left(\frac{l_2}{l_1}\right)$ $\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.9^\circ$

Q15. Adjust the length of vector 2 until it is the same as the length of vector 1, but keep $\theta_1 = 0^\circ$ and $\theta_2 = 90^\circ$. Observe the magnitude and direction of the resultant vector. **Show** how the **magnitude** and **direction** are calculated using the equations you identified above.

Sample Answer: $R = \sqrt{4^2 + 4^2} = 5.66$ units $\theta = \tan^{-1}\left(\frac{4}{4}\right) = 45^\circ$



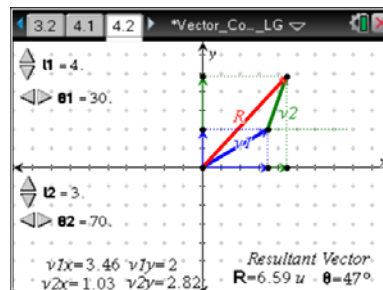
Move to page 4.1.

4. Vectors that are not parallel or perpendicular to each other can be added by using their components.

Read the background information on page 4.1 and then examine the vector diagram on page 4.2.

Move to page 4.2.

Have students answer the questions on the activity sheet.



Q16. Set vector 1 to 4 units at 20°. Set vector 2 to 3 units at 70°. Record the x and y components of each vector below and then add each column to get the x and y components of the resultant. Calculate the **magnitude** and **direction** of the resultant vector using the x and y components. **Show** your work below. Verify that your calculation matches the magnitude and direction displayed in the simulation.

Answer:

Vector	x component	y component
4 units at 20°	3.76	1.37
3 units at 70°	1.03	2.82
Resultant Vector	4.79	4.19

Show your work.

$$R = \sqrt{4.79^2 + 4.19^2} = 6.36$$

$$\theta = \tan^{-1}\left(\frac{4.19}{4.79}\right) = 41.2^\circ$$

Q17. Set vector 1 to 3.5 units at 30°. Set vector 2 to 2 units at 145°. Record the x and y components of each vector below and then add each column to get the x and y components of the resultant. Calculate the **magnitude** and **direction** of the resultant using. **Show** your work below. Verify that your calculation matches the magnitude and direction displayed on the simulation.

Answer:

Vector	x component	y component
3.5 units at 30°	3.03	1.75
2 units at 145°	-1.64	1.15
Resultant Vector	1.39	2.90

Show your work.

$$R = \sqrt{1.39^2 + 2.90^2} = 3.21$$

$$\theta = \tan^{-1}\left(\frac{2.90}{1.39}\right) = 64.3^\circ$$



Q18. Make up your own vector addition problem. Complete the table. Calculate the magnitude and direction of the resultant vector. Show your work. Verify that your calculation matches the magnitude and direction displayed on the simulation page. (Note: If the x component of the resultant vector is negative, you will need to add 180° to your calculated angle to match the direction.)

Answer: The calculations will vary based on the choices of the students.

Note on angles: If the x component of the resultant vector is negative (the resultant vector is in the 2nd or 3rd quadrant), students will need to add 180° to the angle they calculate. They must use

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ in order to match the direction given in the simulation.

TI-Nspire Navigator Opportunities

With TI-Nspire Navigator students can take turns demonstrating their problems using Live Presenter. Throughout the activity, discuss the content with students using Slide Show.

Wrap Up

When students are finished with the activity, collect their handouts. Discuss activity questions using Slide Show.

Assessment

- Formative assessment will consist of questions on the student handout. The Slide Show will be utilized to give students immediate feedback on their assessment.
- Summative assessment will consist of questions/problems on the chapter test.