

# How Long

## Student Activity

7 8 9 **10** 11 12



## Problem

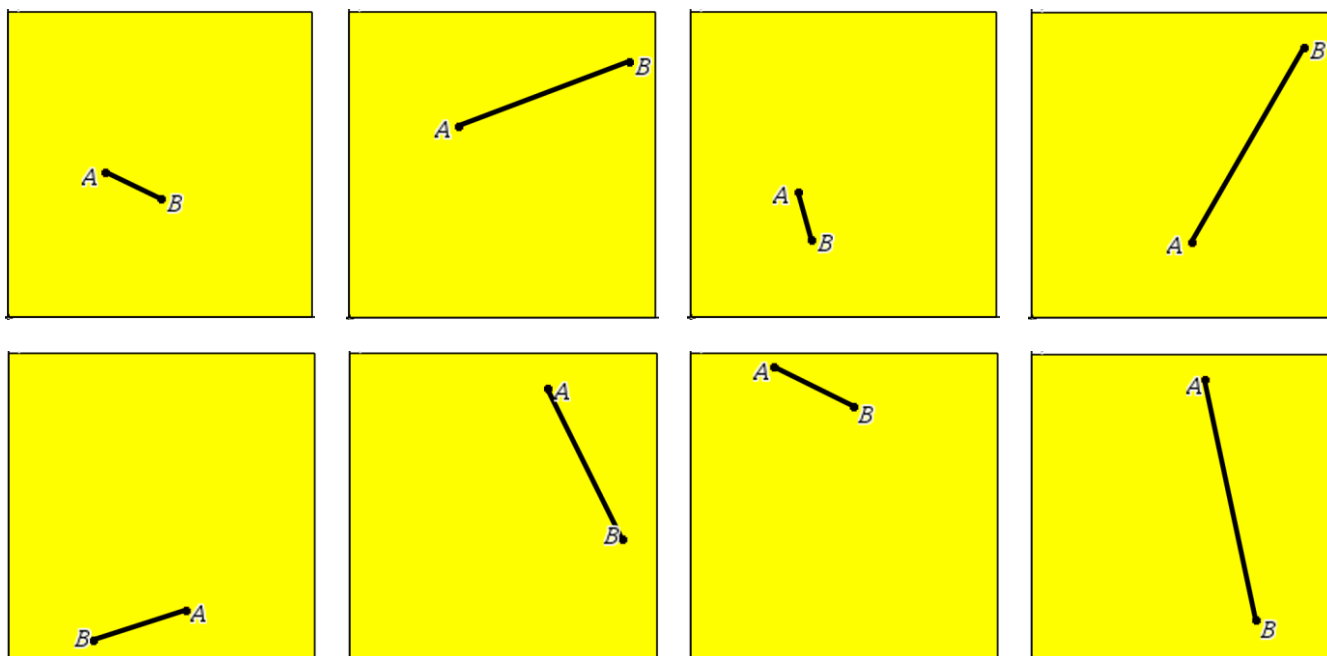
What is the average distance between two points selected randomly within a square with side lengths of 1 unit? To solve this question using analytic methods would require mathematical techniques well beyond high school mathematics courses; however this problem can be solved approximately using simulations, the greater the number of simulations the greater the accuracy of the result. The purpose of this activity is to illustrate how solutions to such problems can be reduced into much simpler versions using simulations.

### Question: 1.

Based on the information above estimate the average length of a line drawn between two random points located inside a square with side lengths 1cm.

### Question: 2.

To begin here are eight samples where two points A and B have been randomly generated so that they land inside the square. The square has been scaled up to 4cm  $\times$  4cm to make it easier to measure the length of line segment AB. Measure the length of the line AB and divide it by 4 to scale it back down, record the result for each square and then compute the average length.



### Question: 3.

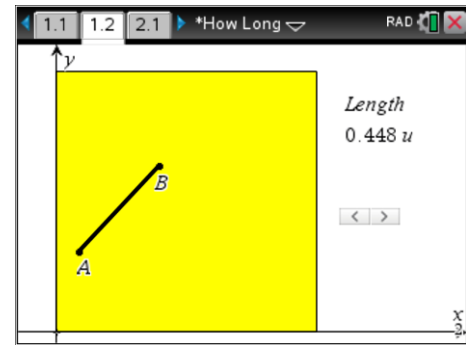
The minimum length of a line segment produced using this method would be 0cm. What would be the longest possible length?

## Using a Simulator

Open the TI-Nspire file: **How Long**

Navigate to page 1.2 where a simulation of the random line segment exists.

Clicking on the spinner will generate a new line and automatically measure the length.

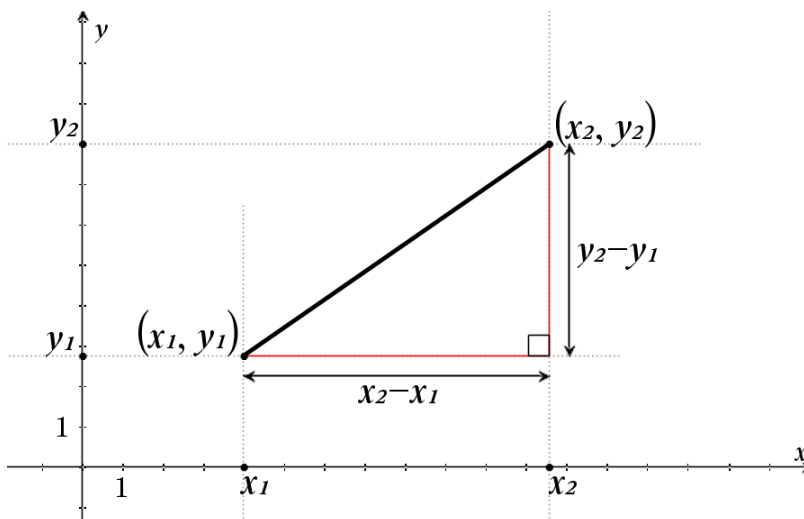


### Question: 4.

Use the interactive diagram on Page 1.2 to record 20 lengths and calculate the average of these lengths.

## Creating Samples

The simulation on the calculator uses the Cartesian plane as a platform. Two points are plotted making it easy to produce random points. The  $\text{rand}()$  command produces a random number between 0 and 1, so two points can be generated by plotting  $x_1 = \text{rand}()$ ,  $y_1 = \text{rand}()$  and  $x_2 = \text{rand}()$ ,  $y_2 = \text{rand}()$ . The length of the line can then be calculated by computing the distance between the two points.



### Question: 5.

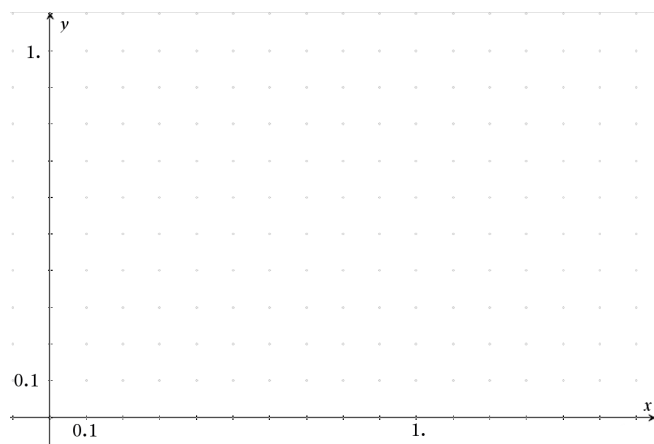
Use the diagram shown opposite and Pythagoras's theorem to derive a formula for the distance between two points.

### Question: 6.

Use the scratchpad to generate 4 random numbers corresponding to  $x_1$ ,  $y_1$ ,  $x_2$ ,  $y_2$ . Record the points and plot them on the Cartesian plane.

Use your formula to calculate the distance between the two points.

Verify your answer by measuring the distance and using the scale to determine the actual distance.



## Creating a LARGE sample

Navigate to page 2.1. A spreadsheet contains five lists:

$$x_1, y_1, x_2, y_2, \text{Dist}$$

In cell A1 type the formula:

$$= \text{rand}()$$

Place the same formula in cells: B1, C1 and D1.

Cell formulas can be duplicated into multiple rows by ‘filling’ the formula down the page.

Navigate to cell A1, then from the menu select **Data > Fill**

Use the navigation pad (down arrow) to move down to cell A50 then press **[Enter]**. Random numbers between 0 and 1 will populate cells A1 through to A50.

Repeat this process for columns B, C and D.

Column E needs to contain the formula for the distance between two points. This formula will be placed at the top of the list in the equation bar to that all values added to the list will automatically be included and calculated. The location for the formula has been highlighted in the screen opposite. Make sure it starts with an equals ‘=’ sign.

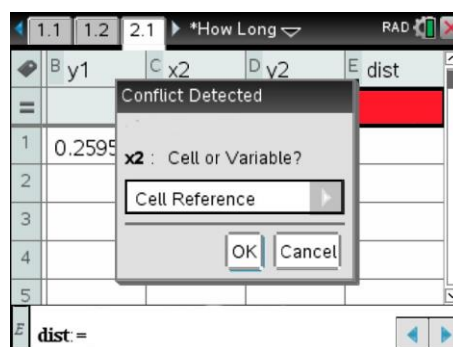
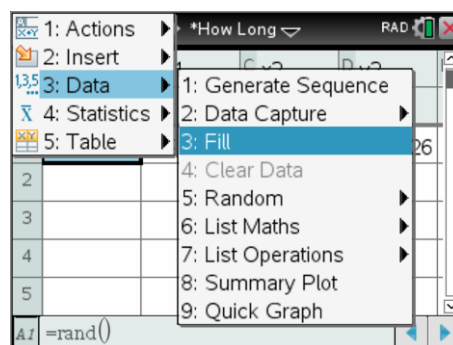
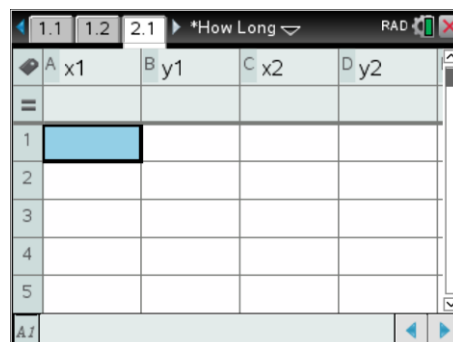
Once your formula is complete you will be advised of a ‘conflict’. This refers to the use of  $x_1, y_1 \dots$  in your formula. The calculator needs to know if you are referring to cell locations (Cell Reference) or a variable you have created. (List name in this case.)

Select **Variable Reference** for each variable in your formula.

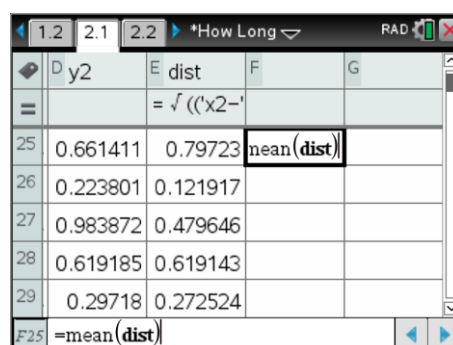
Navigate to cell F1. Type the formula:

$$= \text{mean}(\text{dist})$$

Press **CTRL + R** to regenerate random numbers in all the cells, the calculator will automatically recompute all the distances and determine the mean. (average)



NOTE: The above image has been modified to hide the formula and highlight the location where the column formula is to be located.



### Question: 7.

Record the results of 10 sample means. Use this data to estimate the mean length of a line randomly placed inside a  $1\text{cm} \times 1\text{cm}$  square.

**Question: 8.**

Suppose the side lengths of the square are doubled in length. What do you think the average length of a line placed inside such a square would be?

**Question: 9.**

Adjust all relevant formulas in the spreadsheet and record the mean for 10 samples. Use this data to determine the mean length of a line randomly generated inside a 2cm x 2cm square.

Compare your results with your estimate from the previous question.

**HINT:**  $2 \times \text{rand}()$  will produce random numbers between 0 and 2.

**Question: 10.**

Conduct an investigation into the average length of a line placed inside a rectangle with dimensions 1cm x 2cm.

**Extension Investigation**

Suppose two random points are placed inside a box measuring 1cm x 1cm x 1cm. What would be the average length of the line connecting these two points?

- Set up the spreadsheet to produce two random points:  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$ .
- Adjust your distance formula to measure the distance between two points in three dimensions.
- Collect and record a sample set of results

**Thinking:** What would this problem look like in four dimensions? Think of the fourth dimension as time, and two objects moving around randomly in a confined space. What would be the average distance between these two objects?