Radians

Teacher Notes & Answers

7 8 9 10 **11** 12

TI-Nspire™ Investigation Student 30 min

Introduction

Have you ever wondered why there are 360 degrees in a circle? What if we tried to make a 'metric' circle. For a metric circle we could argue for either 100 or 400 degrees, depending on whether we focus on a full revolution (100) or we consider 'right angles' (100) as the dominant reference. As it turns out this measure is already in existence, the measure is referred to as "Gradians". Another interesting consideration represents a movement away from rational numbers. As π is already entrenched in everything circle related, why not measure angles in terms of π ? As it turns out, this measurement is much more useful than degree and gradian measures.

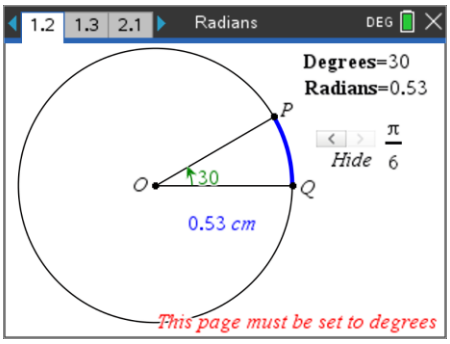
Introduction

Open the TI-Nspire document: **Radians**

Navigate to page 1.2.

A circle of radius one unit is displayed. Point P can be dragged around the circle. Angle ∠QOP is measured in degrees, the arc length is displayed in centimetres (cm). The "Radian" measure will be exactly the same as the arc length for the unit circle.

An exact measurement for the arc length can be displayed or hidden.



Question: 1

What is the circumference of the circle?

Answer: 2π

Question: 2

Position point P to determine the radian measure for 180°?

Answer: π

Question: 3

The exact radian measure for 30° is displayed as $\dfrac{\pi}{6}$, show how this result might be calculated by hand.

[**Hint**: Refer to Question 2]

Answer: Answers will vary. Students should note that $180° = \pi^r$ and that 30/180 = 1/6 therefore $30° = \pi/6^r$.

This may also be approached as: $180° = \pi^r$ … divide both sides by 6.

Question: 4

Complete the table below for the conversion between degrees and radians.

Degrees	0	30	45	60	90	120	135	150	180
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π

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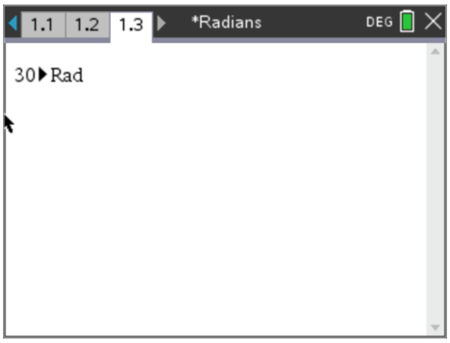


The document is currently set to degrees. [Top right corner of screen]

Navigate to page 1.3, a Calculator application. Type the value: 30. As the calculator is currently set to "DEG", the value will be assumed as 30° if it represents an angle.

Use the catalogue key: [catalogue key] and press R, this will navigate to menu items starting with the letter R.

Navigate to ▶**RAD**

Press [enter] to accept the command and [enter] again to execute.

Question: 5

What answer does the calculator provide when converting 30° to radians? [Include any symbols]

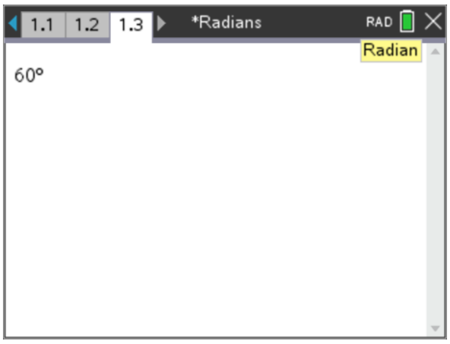
Answer: Students should note that the radians symbol is included in the answer: $\pi/6^r$. Students should **not** write: $30 = \pi/6$ as this is very different than $30° = (\pi/6)^r$.

Question: 6

When the calculator is in degree mode it will automatically covert a radian angle into degrees. Enter $(\pi/4)$ and use the symbols template to add the radian symbol.

Note: The symbols template can be obtained by pressing: [ctrl] + [catalogue key]

Answer: Students must enter $(\pi/4)^r$ to get 45 degrees. If students enter $\pi/4^r$ then the calculator will assume that 4 is the only portion of the number measured in radians.



The document is currently set to degree mode. The mode can be changed via the document settings, or on the TI-Nspire CX II CAS, you can change the mode for the current application by simply clicking on the DEG symbol.

Change the document to RAD (Radians)

Enter an angle of 60°. Use the symbols template to access the degree symbol or the punctuation fly-out menu. When using the fly-out menu, you can continue to tap the key to scroll through the menu.

Question: 7

Convert 150° to radians using the calculator and explain how the answer relates to 30° and $\pi/6$.

Answer: $5\pi / 6$. Since $150 = 5 \times 30$ then it follows that $5 \times \pi/6$ will be $5\pi/6$.

Question: 8

Convert 210° to radians using the calculator and explain how the answer relates to 30° and $\pi/6$.

Answer: $7\pi / 6$. Since $210 = 7 \times 30$ then it follows that $7 \times \pi/6$ will be $7\pi/6$.

Question: 9

Convert 225° to radians using the calculator and explain how the answer relates to 45° and $\pi/4$.

Answer: $5\pi / 4$. Since $225 = 5 \times 45$ then it follows that $5 \times \pi/4$ will be $5\pi/4$.

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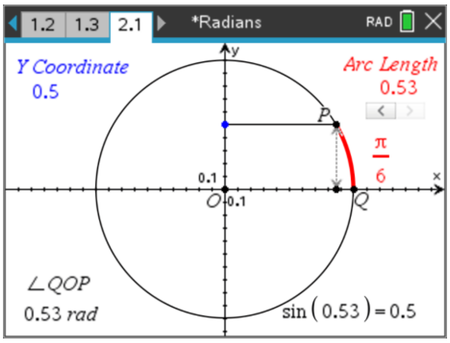
Why?

Why do mathematicians bother with radians when degrees seem much easier? A clue lies in the trigonometric functions. Navigate to page 2.1 and once again, drag point P around the unit circle. This time the unit circle has been place on the Cartesian plane. We are interested in two measurements:

- The length of the arc from Q to P.
- The distance from P to the x axis. (Y coordinate)

Note: The exact value for the arc length can be toggled off / on.



Question: 10

The diagram above shows sin(0.53) = ½ or sin(π / 6) = ½. Find another arc length where this is also true.

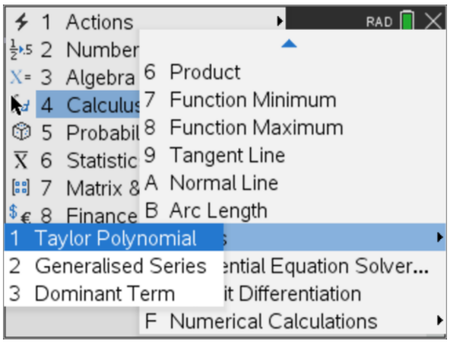
Answer: Alternative arc length is at 5π/6.

Question: 11

Find two arc length where the y coordinate is equal to –½. Comment on your findings.

Answer: Two arc lengths: 7π/6 and 11π/6. Students may notice that these are both π/6 units below the x axis.

The radian measure and its relationship with circular functions go much further.



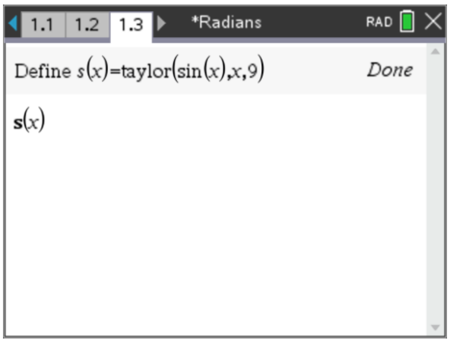
Navigate back to page 1.3.

Define a function s(x), this will be a special type of polynomial.

menu > **Calculus** > **Series** > **Taylor Polynomial**

The syntax for the **Taylor** polynomial is:

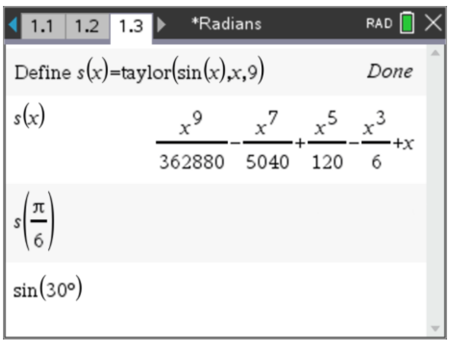
Taylor (expression, variable, degree)



The expression being investigated is: sin(x), the variable is x and degree of the polynomial is 9. [It can be much higher, but this will do for now.]

Once the expression has been defined type: s(x)

Determine the value of s(π/6), press ctrl + enter to obtain an approximate answer.



Compare the result to sin(π/6) and sin(30°), remember to use the degree symbol since the calculator should now be in radian mode.

Note: The answers have been removed from the screen opposite.

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Question: 12

How do the answers compare?

Answer: All the answers are the same (approximately).

Question: 13

Compare answers for s(π/3), sin(π/3) and sin(60°).

Answer: The three results are (approximately) the same ≈ 0.8660.

The Taylor polynomial is a function of infinite degree:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} \ldots$$

Question: 14

What would be the next term in this sequence?

Answer: $x^{13} \div 13!$

Question: 15

Using $x = \frac{\pi}{6}$ explain why the next term (Question 10), and subsequent terms are not critical for small angles.

Answer: The next term in the sequence ≈ 0.000000000020316 and therefore makes very little difference to the result.

Question: 16

Determine the Taylor polynomial for cos(x) and test some angles. [Make sure you are using radians.]

Answer: Taylor polynomial: $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!}$. Angle trials will vary but will all concur with cos(x).

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