Math Objectives

- Students will be able to identify the domain and range of step functions.

Vocabulary

- domain
- function
- open points on graph

- range
- step function
- closed points on graph

About the Lesson

- In this activity, students will learn to identify the domain and range of various real-world step functions. They will graphically explore numerical data points and observe step functions. Open and closed points on a graph are investigated and discussed. Students use self-check questions to check their understandings with immediate feedback. Extension questions are also provided to apply what is learned to determine the domain and range of other functions.

TI-Nspire™ Navigator™ System

- Use Class Capture to check students' reasoning about a range of intervals and to explore multiple solutions.
- Use Quick Poll to assess student understanding of the domain and range

Activity Materials

Compatible TI Technologies: 📱 TI-Nspire™ CX Handhelds, 👆 TI-Nspire™ Apps for iPad®, 💻 TI-Nspire™ Software
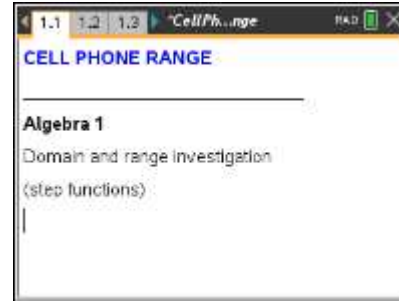
Tech Tips:

- This lesson includes screen captures taken from the TI-Nspire CX II handheld. It is also appropriate for use with the TI-Nspire family of products, including TI-Nspire CX software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity

- CellPhoneRange_Student.pdf
- CellPhoneRange_Student.doc

TI-Nspire document

- CellPhoneRange.tns

Discussion Points and Possible Answers

> **Tech Tips:**
>
> It would be beneficial for students to be familiar with navigating between page (ctrl + ◀ *or* ctrl + ▶), toggling between apps (ctrl + tab) and using self-check questions. These questions can be checked by using ctrl + ▲ *or MENU > Check Answer.*
>
> Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity, but the use of the Navigator System is not necessary for the completion of this activity.

Problem 1 – Cell Phone Situation

The definition for domain and range is explained.

Move to page 1.2.

1. The **domain** is the set of _____ where the function is defined.

    Answer: inputs

Move to pages 1.3 – 1.4

The cell phone situation is explained to be 40 cents per minute with a limitation of 4 minutes per call.

2. Your friend got a cell phone that charges 40 cents per minute (or any part of a minute). However, he is not allowed to use it for more than 4 minutes per call.
    a. If he talks for 0.5 seconds, how much was the fee?

        Answer: The fee for 0.5 seconds is 40 cents (written as $0.40). It is important for students to realize that $0.40 is not the same as 0.40 cents.

    b. If the cost is a function of time, what is the domain of the cost function?

        Answer: The domain is 0 < time ≤ 240 sec.

Move to page 1.5

3. The output of a function is called the _____.

    Answer: range
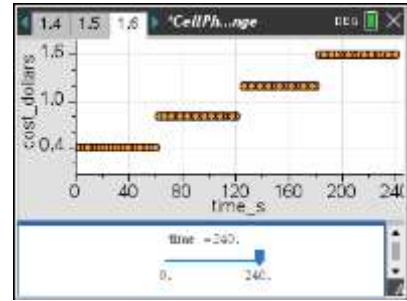
Move to pages 1.6 and 1.7.

Points are plotted using the slider below the graph and an automatic data capture.

On page 1.6, move the slider to change the amount of time that your friend talks on the costly cell phone. Observe the output.

4. Describe the graph.

   Answer: As the students drag the slider, data points will be added to the graph above in the *Data & Statistics* application.

5. What is the **range** of the cost function?

   Answer: The range is {$0.40, $0.80, $1.20, $1.60}

Move to pages 1.8, 1.9, and 1.10

On page 1.9, use **MENU > Trace > Graph Trace** to explore the function. When using **Graph Trace**, you can type in a number to jump to that value.

6. On the graph on page 1.9, why so some of the points have open circles and other points closed or filled in?

   Answer: Closed points are where the function is defined; open points mean the function in not defined there.
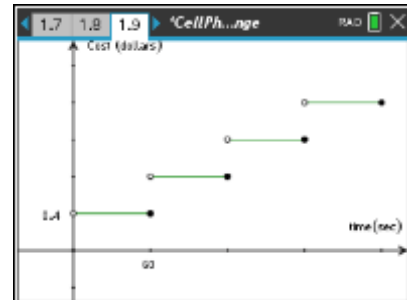
7. _____ points occur where the function is _____.

   Answer: Closed, defined OR Open, undefined

TI-Nspire Navigator Opportunity: Class Capture and/or Live Presenter
Throughout the lesson, use Class Capture to verify students have entered the correct answers to the questions. Page 1.6 is a good place to use Class Capture to display student data sets. Scroll through the different student results. Students should see that the data is not continuous because it "jumps" from one value to the next.
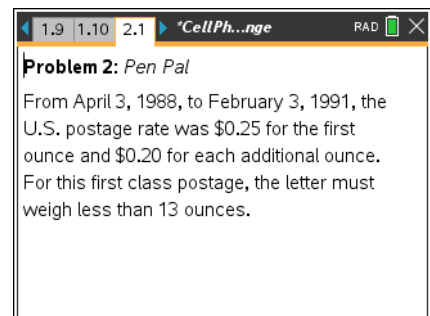
Problem 2 – Snail Mail "Pen Pal" Situation

Pages 2.1 – 2.5

From April 3, 1988, to February 3, 1991, the U.S. postage first class postage rate was $0.25 for the first ounce and $0.20 for each additional ounce. A letter, using first class postage, must weigh less than 13 ounces.

8. The cost is a function of the weight. What is the **domain** of this function? Let x be the variable for the input.

   Answer: The domain is {x: $0 < x < 13$}.

9. Using the graph on page 2.3, determine how much it cost to mail a 7.25 ounce letter.

Answer: A 7.25 ounce letter costs $1.65.

10. The graph on page 2.3 represents what type of function?

   Answer: Step Function

11. What is the **range** of this function?

   **Answer:** A range of {y: y = 0.25, 0.45, 0.65, 0.85, 1.05, 1.25, 1.45, 1.65, 1.85, 2.05, 2.25, 2.45, 2.65}.

TI-Nspire Navigator Opportunity: Quick Poll
You may want to use Quick Poll to verify students understand the concepts of domain and range. The questions provided in the Extension/Homework assignments may be used for this purpose.

Extension/Homework #1 – Shipping Situation

The students are given the situation where the shipping cost is $11.45 for the first pound and $0.60 more for each additional pound.

Students can press ⌨ to use the **Scratchpad** to make their calculations. Press esc to return to the lesson.
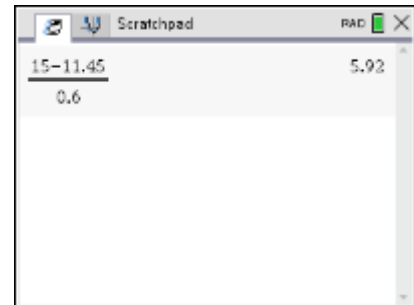
Pages 3.1 – 3.3

Your friend calls you on your cell to tell you about a great sale at an online store for some supplies you need for a school function. You find out that the shipping cost to get it in 2 days is $11.45 for the first pound and $0.60 more for each additional pound.

12. You decide you can only spend $15.00 for shipping. With this limitation, what is the **domain** of the cost function for shipping a package?
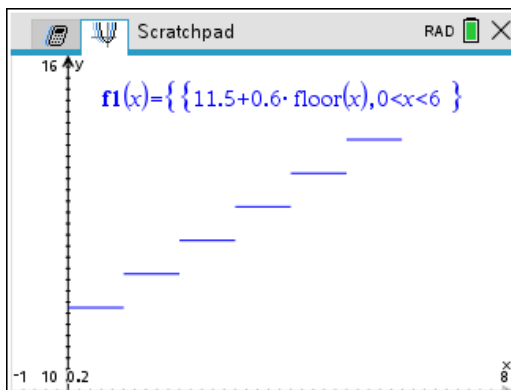
   Answer: Domain is {x: 0 < x < 6}

13. What is the **range** for this cost of shipping function?

   Answer: The range for this function that again looks like a step function is
   {y: y = 11.45, 12.05, 12.65, 13.25, 13.85, 14.45}.

14. Make a sketch of what the graph would look like.

   Answer:

To make this occur on the calculator, one must type {11.5 + 0.6*floor(x)|0<x<6}

Extension/Homework #2 – Functions

Various functions are considered. The domain is restricted by the allowable inputs. Dividing by zero and taking the square root of negative numbers are not allowed. A graph of these functions can be helpful by allowing students to see that the domain can be limited. Similarly, have students graph the function and ask what output values occur for functions like $y = x^2$.

Pages 3.4 – 3.9

For the function y = 2x, the domain is all real numbers or {x: x ∈ ℝ}. The output, or range, is similarly the set of y such that y is an element of the reals.

Consider the following questions and determine the **domain** and **range**. Be especially mindful of dividing by zero or taking the square root of negative numbers. If needed, you may graph the function to confirm and to deepen your understanding of the solution.

15. $y = x^2$

    Domain: _____   Range: _____

    Answer: domain: {x: x ∈ ℝ}, **range**: {y: y ≥ 0}

16. $y = \sqrt{x}$

    Domain: _____   Range: _____

    Answer: domain: {x: x ≥ 0}, **range**: {y: y ≥ 0}

17. $y = \dfrac{5}{x}$

    Domain: _____   Range: _____

    Answer: domain: {x: x ≠ 0}, **range**: {y: y ≠ 0}

18. $y = \dfrac{3}{x+5}$

    Domain: _____   Range: _____

    Answer: domain: {x: x ≠ –5}, **range**: {y: y ≠ 0}

19. $y = \sqrt{2x - 6}$

Domain: _____   Range: _____

Answer: domain: $\{x : x \geq 3\}$, **range**: $\{y : y \geq 0\}$