

# Six Trigonometric Functions

## Student Activity

7 8 9 10 11 12



## Introduction

The three trigonometric functions taught most often in high school are: sine (sin), cosine (cos) and tangent (tan). There are however three other related trigonometric functions secant (sec), cosecant (cosec) and cotangent (cot). The origins of these terms can be attributed to Arabic mathematicians. The word 'sine' however comes from a mistranslation of the word 'chord'. In the unit circle sine is the measure of a semi-chord (half chord). Other terms such as cosine, cotangent and cosecant are all  $90^\circ$  out of phase with sine, tangent and secant as the prefix 'co' relates to 'complement'.

$$\cos(\theta) = \sin(90^\circ - \theta)$$

$$\cot(\theta) = \tan(90^\circ - \theta)$$

$$\csc(\theta) = \sec(90^\circ - \theta)$$

Imagine your calculator had just one trigonometric ratio, let's say: 'sine'. It is possible to use this one ratio to generate all the others! We can see from above that cosine is easy.

### Example:

Suppose  $\theta = 60^\circ$ , then according to the above relationships:  $\cos(60^\circ) = \sin(90^\circ - 60^\circ) = \sin(30^\circ)$

## Similar Triangles

Open the TI-Nspire file "Six Trig Functions".

Page 1.2 contains a unit circle that can be used to explore all six trigonometric ratios. The example shown opposite is for cosine, this ratio is read from the x axis.

Use the slider beneath 'now showing' to explore each of the ratios on the unit circle. Drag point P around the circle to see how each one is calculated based on the size of the angle.

Navigate to Page 2.1 and select "Problem 1" using the slider.

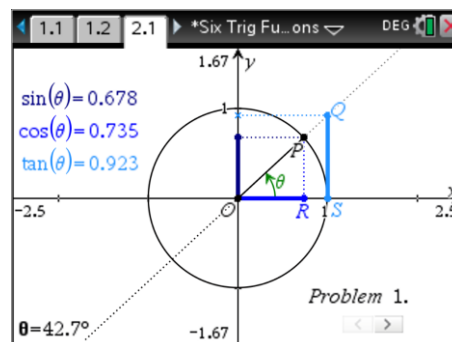
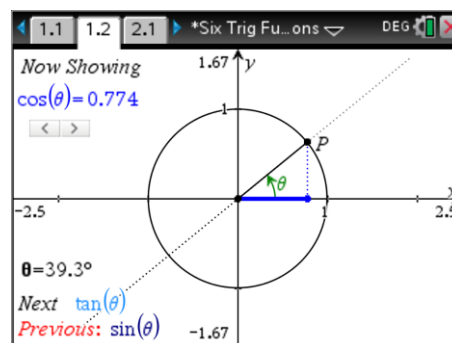
Triangle ORP has sides OR (cosine), RP (sine) and OP (1 unit).

Triangle OSQ has sides OS (1 unit), SQ (tangent) and OQ (TBD)<sup>1</sup>

Angle POR is measured by  $\theta$  and is common to both triangles.

Angle ORP and OSQ are both  $90^\circ$

So by AAA, triangles ORP and OSQ are similar.



### Question: 1

Use the similar triangles ORP and OSQ to show that  $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

<sup>1</sup> TBD = To be determined (This length is determine later in this activity)

**Question: 2**

The following exact values are known for  $\sin(\theta)$ :

$$\sin(30) = \frac{1}{2}$$

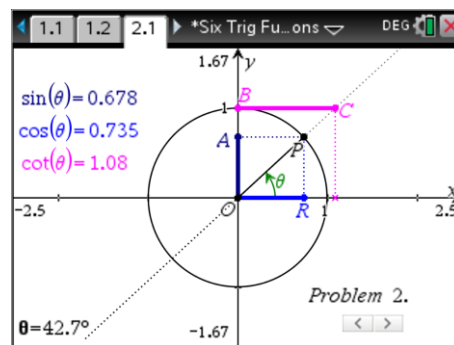
$$\sin(45) = \frac{\sqrt{2}}{2}$$

$$\sin(60) = \frac{\sqrt{3}}{2}$$

Use these exact values and the established relationships between sine, cosine and tangent to determine the corresponding exact values for cosine and tangent.

On Page 2.1 select “Problem 3” using the slider.

Triangle OBC and OAP are similar. This similarity can be used to establish a relationship between  $\cot(\theta)$ ,  $\sin(\theta)$  and  $\cos(\theta)$  or collectively,  $\tan(\theta)$ .

**Question: 3**

Use the similarity of triangles OBC and OAP to write a rule relating  $\cot(\theta)$ ,  $\sin(\theta)$  and  $\cos(\theta)$ , and hence express  $\cot(\theta)$  in terms of  $\sin(\theta)$  only.

**Question: 4**

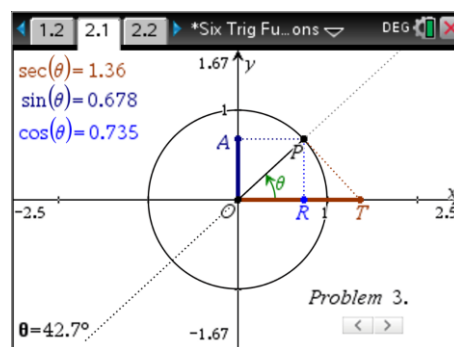
Use the exact values for  $\sin(\theta)$  to calculate exact values for  $\cot(30)$ ,  $\cot(45)$  and  $\cot(60)$ .

**Question: 5**

Use the similarity of triangles OBC and OAP to write a rule relating  $\cot(\theta)$  to  $\tan(\theta)$ .

On Page 2.1 select “Problem 3” using the slider.

Triangles OPR and OPT are similar as they both contain a right angle and have  $\theta$  in common. This similarity can be used to establish a relationship between  $\sec(\theta)$ ,  $\sin(\theta)$  and  $\cos(\theta)$ .

**Question: 6**

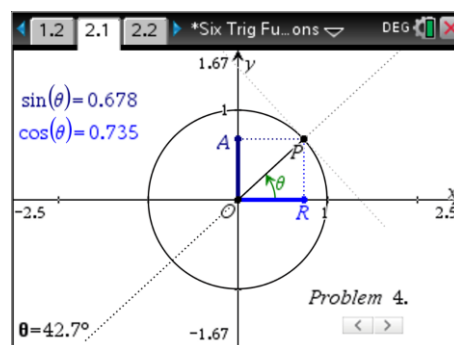
Use the similarity of triangles OPR and OPT to write a rule relating  $\sec(\theta)$  to  $\cos(\theta)$  and hence write exact values for  $\sec(30)$ ,  $\sec(45)$  and  $\sec(60)$ .

**Question: 7**

Use the similarity of triangles OPR and OPT to show that length  $PT = \tan(\theta)$ .

## Pythagoras Rules

Pythagoras's theorem can also be used to develop relationships between trigonometric functions. The following collection of problems relies on Pythagoras's theorem.



### Question: 8

Use the slider to display information for problem 4. Use Pythagoras's theorem to write a relationship between  $\sin(\theta)$  and  $\cos(\theta)$ . Check your answer using a range of angles.

### Question: 9

Use the slider to return to problem 3 and show that:  $1 + \tan^2(\theta) = \sec^2(\theta)$ .

**Hint:** Use your result from Question 7.

### Question: 10

Use the slider to return to problem 2, use similar triangles to show that length OC is equal to  $\operatorname{cosec}(\theta)$  and hence show that:  $1 + \cot^2(\theta) = \operatorname{csc}^2(\theta)$ .

### Question: 11

Navigate to page 2.2 (calculator application) and enter the equation:  $1 + \cot^2(\theta) = \operatorname{csc}^2(\theta)$ .

How does the calculator respond? Discuss.

### Question: 12

Select Problem 3 on page 2.1. From Question 7 we know that  $PT = \tan(\theta)$ . We can also see that  $PR = \sin(\theta)$  and that  $RT = \sec(\theta) - \cos(\theta)$ . From Pythagoras's theorem we can therefore conclude:

$$(\sec(\theta) - \cos(\theta))^2 + \sin^2(\theta) = \tan^2(\theta)$$

Using the result from question 7, make an appropriate substitution for  $\sec(\theta)$  and show (by hand) that the above equation is true.