Math Objectives

- Students will determine and analyze a polynomial model for a section of roller coaster track.
- Students will utilize translations to adjust their model to fit various criteria.
- Students will construct a cubic equation in factored form to fit given criteria.
- Students will apply mathematics to solve problems arising in everyday life, society, and the workplace (CCSS Mathematics Practice).
- Students will interpret mathematical results in the context of the situation and reflect on whether the results make sense, improving the model as necessary (CCSS Mathematics Practice).
- Students will use different forms for algebraic expressions (CCSS Mathematics Practice).
- Students will use technological tools to explore and deepen understanding of concepts (CCSS Mathematics Practice).

Vocabulary

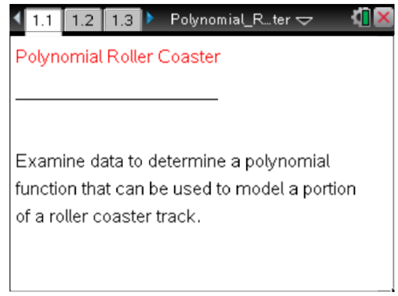
- cubic regression
- degree of a polynomial equation

About the Lesson

- This lesson involves finding a cubic regression equation to model a section of roller coaster track.
- As a result, students will:
  - See the effect that various transformations have on the graph of a function.
  - Analyze the regression equation to answer various questions.
  - Adjust their equations to fit various criteria, and understand the advantages of writing them in various forms.
  - Write a cubic equation, in factored form, to fit given criteria.

TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine patterns that emerge and monitor students' understanding.
- Use Live Presenter to provide assistance to students throughout the activity.
- Use Quick Poll to assess students' understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- Once a function has been graphed, the entry line can be shown by pressing [ctrl] [G]. The entry line can also be expanded or collapsed by clicking on the chevron.

Lesson Files:

Student Activity
Polynomial_Roller_Coaster_Student.pdf
Polynomial_Roller_Coaster_Student.doc

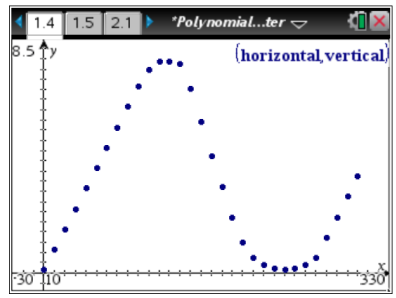
TI-Nspire document
Polynomial_Roller_Coaster.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

The *Ride of Steel* roller coaster in Darien Lake, New York, reaches a maximum height of about 208 feet.[1] Page 1.3 contains a spreadsheet of data of horizontal and vertical distances, in feet, of a section of roller coaster track. We will examine the data for this segment of the roller coaster and find a polynomial function to model the track.

Move to page 1.4.

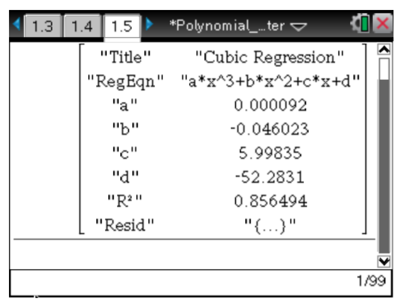


1. What is the lowest degree polynomial function that could model the points in the scatter plot on Page 1.4? Explain your answer.

   Answer: Since the scatter plot has two extrema, it can be modeled by a polynomial of degree 3 or greater.

> **Teacher Tip:** The data for this problem are not actual data values, but have been generalized from visual reference.

Move to page 1.5.



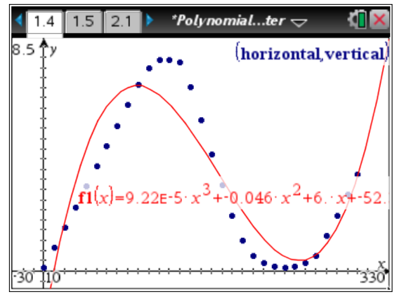
2. Use this Calculator page to find a cubic regression equation. Select **MENU > Statistics > Stat Calculations > Cubic Regression**. Select horizontal for the X List, vertical for the Y List, and save it to **f**1. Press OK or enter.

> **Teacher Tip:** If students are not familiar with obtaining a regression equation to model the data, be sure to progress slowly through this section.

TI-Nspire Navigator Opportunity: ***Screen Capture and Live Presenter***
See Note 1 at the end of this lesson.

[1] http://godarienlake.com/rides/thrill-rides/

Move back to page 1.4.



3. To activate, or graph, the regression equation just found, select **MENU > Graph Entry/Edit > Function**. Move to **f**1(*x*), press **enter**, and the equation will be activated, or graphed. Write the regression equation below.

   Answer: The regression equation is
   $f(x) = 0.000092x^3 - 0.046023x^2 + 5.99835x - 52.2831.$

   > **Tech Tip:** The Calculator page (Page 1.5) displays the coefficients to more decimal places than the graphing screen shown on Page 1.4.

4. According to the regression equation, what is the maximum height you could expect to reach on this portion of the roller coaster track? According to the data, what is the maximum height you could expect to reach on this portion of the roller coaster track? Explain why the two values are different.

   Answer: According to the regression equation, the maximum height is 182 feet. According to the data, the maximum height is 208 feet. The two values are different because the regression equation is only a best fit model to the data. The graph of the regression equation does not necessarily pass through the same maximum or minimum points on the scatter plot.

   > **Tech Tip:** To find the maximum height of the function, select **MENU > Analyze Graph > Maximum**. Select the lower and upper bounds.

5. Based on the regression equation, when you've traveled 500 feet horizontally, what height could you expect to reach? Does this value make sense? Explain.

   Answer: To determine the height, find f1(500). Based on the equation, we could expect to reach a height of 2961.22 feet by traveling 500 feet horizontally. Such a height is not plausible for a roller coaster.

   > **Tech Tip:** You might want to suggest that that students utilize their Calculator page and enter f1(500) to obtain this answer.

6. What does your answer to question #5 tell you about the equation that you obtained?

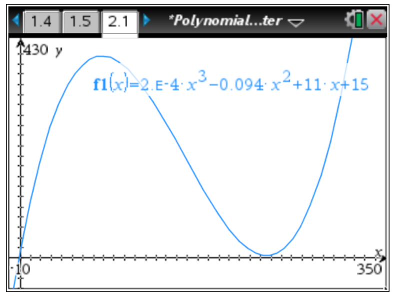
   Sample Answers: Apparently, our model is not very good at that horizontal distance. After a cubic function reaches its minimum value, it will continue to increase without bound. However, a roller coaster will have to reach a maximum and then return to the boarding platform.

We might want to model the data with a higher degree polynomial that has an additional maximum value. Alternatively, a piecewise defined function might be used to fit various sections of the roller coaster track.

7. A cubic function can have three zeroes. Would you expect a function that fits this data to have three zeroes? Explain.

Answer: If this function had three real zeros, the passengers would board on ground level, at the first zero. The roller coaster would have to go below ground for part of the time, at the second zero, and then return to ground level, at the third zero.

Move to page 2.1.



The Kingda Ka Flags Great Adventure in Jackson, New Jersey, with a drop of 418 feet, is reported to be the world's tallest roller coaster.[2] Page 2.1 contains the graph of a section of a polynomial roller coaster function with equation $y = 0.0002x^3 - 0.094x^2 + 11x + 15.$

Teacher Tip: You might want to remind students that one story is approximately 10 feet. This will help them envision the heights discussed.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 2 at the end of this lesson.

8. What is the maximum height that the roller coaster reaches in this section? What is the minimum height?

Answer: The roller coaster reaches a maximum height of approximately 396 feet, and a minimum height of approximately 4.41 feet.

9. Is the roller coaster boarded at ground level? If not, from what height is it boarded? Is this height reasonable?

Answer: Since **f**1(0) = 15, the roller coaster is boarded 15 feet above the ground. Thus, passengers must go up approximately 1.5 stories to board the roller coaster.

[2] http://www.sixflags.com/greatadventure/rides/kingdaka.aspx

10. Assume that there was a smaller section of the roller coaster that preceded the section shown in this graph. To add the section of roller coaster from Page 2.1 to the plans for the complete roller coaster, we will have to shift this section 100 feet to the right. Write an equation to do so, and enter your answer into **f**2(x) to check your work.

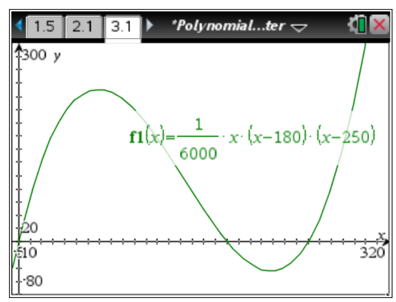
Answer: This shift is a horizontal translation 100 feet to the right, so we can enter the answer as **f**2(x) = **f**1(x - 100).

TI-Nspire Navigator Opportunity: ***Quick Poll or Screen Capture***
See Note 3 at the end of this lesson.

11. The builder of the roller coaster decided that a minimum height of approximately 4.41 feet was too close to the ground and now wants to raise this section of the roller coaster so it is at least 15 feet above the ground. Write an equation that would make this change, and enter your answer into **f**3(x) to check your work.

Answer: To move the roller coaster 15 feet above the ground, it is necessary to make a vertical translation of 15 - 4.41 = 10.59 feet. We can enter the answer as **f**3(x) = **f**1(x) + 10.59.

Move to page 3.1.



It is not difficult to create a polynomial roller coaster as we have done. Create a simple cubic polynomial equation, and then adjust it to model a roller coaster.

12. Write an equation, in factored form, for a cubic polynomial function to model a section of roller coaster with the following conditions:
    - the roller coaster is boarded at ground level,
    - reaches a maximum height,
    - returns to ground level after traveling horizontally for a total of 180 feet,
    - reaches a minimum height,
    - returns to ground level after traveling horizontally for a total of 250 feet,
    - and continues to travel.
    - The coaster reaches a height of 200 feet after traveling 100 feet horizontally.

    Enter your equation into **f**1(x), and copy it below.

    Sample Answers: If the roller coaster were boarded at ground level, the point (0, 0) would lie on the graph. The maximum value of the function would occur between x = 0 and x = 180. The minimum value would occur between x = 180 and x = 250. The point (100, 200) would also lie on the graph.

Since the coaster returns to ground level at x = 180 and x = 250, the three zeros of the function are (0, 0), (180, 0) and (250, 0).

Thus, $y = ax(x-180)(x-250).$

To determine the value of *a*, we substitute the point (100, 200) into the equation and solve for *a*.

$$200 = a(100)(100-180)(100-250)$$

We obtain: $a = \dfrac{1}{6000}$

Our equation is $y = \dfrac{1}{6000}x(x-180)(x-250).$

13. Based on your equation, at what height do the passengers board? What is the maximum height reached by the roller coaster? What is the minimum height?

    Answer: The roller coaster is boarded at ground level. Its maximum height is approximately 231 feet. Its minimum height is approximately -44.2 feet, indicating that the roller coaster goes below ground.

14. Do you think it would be possible to build a roller coaster to fit your model? Why or why not?

    Sample Answers: It might be difficult to construct a roller coaster that goes below ground for a short period of time. However, there are roller coasters that do travel underground. The Voyage Roller Coaster in Santa Claus, Indiana has five underground tunnels.[3].

15. Adjust your equation so the minimum height of the roller coaster is approximately 10 feet. Enter your equation into **f**2(*x*), and copy it below.

    Answer: To move the roller coaster 10 feet above the ground, it is necessary to make a vertical translation of 10 - -44.2 = 54.2 feet. We can enter the answer as **f**2(x) = **f**1(x) + 54.2.

16. If the equation were adjusted as suggested in question #15, what changes have to be made in the location of the boarding platform?

    Answer: Since the entire roller coaster would be moved up 54.2 feet, the boarding platform would now be over five stories high.

[3] http://www.holidayworld.com/rides/voyage

17. How could you create a section of roller coaster with more peaks and dips?

Answer: Use a polynomial function of a higher degree to create a roller coaster with more peaks and dips.

18. How would you determine the lowest degree polynomial function to use for a roller coaster with *n* peaks and dips?

Answer: If there were *n* peaks and dips, the degree of the polynomial function would be at least *n* + 1.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The types of information they can obtain from a polynomial model of a roller coaster.
- The effect of translations on the graphs of polynomial functions.
- How to create a cubic equation in factored form to fit given criteria.

Extension

Have students research roller coasters and challenge them to create polynomial roller coasters using equations with degree greater than three.

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Note 1

Question 2, Name of Feature: Live Presenter and Screen Capture

If students are not familiar with the procedure for obtaining a regression equations, ask one of the students to serve as the *Live Presenter* and demonstrate to the class how to navigate through the activity. You might want to leave *Screen Capture* running in the background, with a 30 second automatic refresh, and without student names displayed, to enable you to monitor students' progress and make the necessary adjustments to your lesson.

Note 2

Question 8, Name of Feature: Quick Poll

You might want to send a *Quick Poll* to assess students' understanding of the answers to the questions in this section.

Note 3

Questions 9 and 10, Name of Feature: Quick Poll and Screen Capture

You might want to send a *Quick Poll* or use *Screen Capture* to see the different forms of the equations for questions 9 and 10.

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