Parametric Races & Slopes

ID: 12516

Time Required
15 minutes

Activity Overview

Students investigate parametric equations. They will study the motion of two points and use calculus to determine the velocity and acceleration. The slope of parametric equations will be determined. As an extension, students will create a graph and find the value of t for the vertical and horizontal tangents and investigate vectors is an optional extension.

Topic: Parametric Equation

- *Parametric equations used to find the investigate velocity and acceleration.*
- *Find the slope from parametric equations. Investigate vectors (optional extension).*

Teacher Preparation and Notes

- *To graph a parametric equation on the TI-Nspire on a Graphs & Geometry page, select **MENU > Graph Type > Parametric**. Alternatively, use 'right-click' (ctrl + menu) when on the entry line.*
- *On self-check questions, students can get immediate feedback by pressing menu **Check Answer** (or ctrl + ▲). With TI-Nspire Teacher Edition software, teachers can change self-check questions to exam mode so students cannot check their answer. On any question click the Teacher Tool Palette and select Question Properties. Change the Document Type from Self-Check to Exam.*
- *Parametric equations are a Calculus BC topic. AB teachers may enjoy using this activity after the AP exam or using with students in your AB class who want to prepare for the BC exam. After completing the activity, students should be more successful with AP* multiple-choice questions like 03BC4,7,17,84, 98BC10,21(arc length- see CalcWeek27 Gateway Arc Length),77, 88BC34, and free-response questions like 05formB BC1, 04BC3, 04 form B BC1, 03BC2, 03formB BC4, 02BC3, 02formB BC1, 01BC1, 00BC4, 99BC1, 97BC1, 95BC1.*
- ***To download the student TI-Nspire documents (.tns file) and student worksheet, go to education.ti.com/exchange and enter "12516" in the quick search box.***

Associated Materials

- *CalcWeek28_ParametricEq_Worksheet_TINspire.doc*
- *CalcWeek28_ParametricEq.tns*

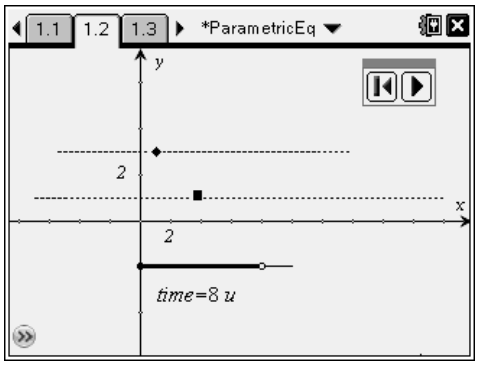
Suggested Related Activities

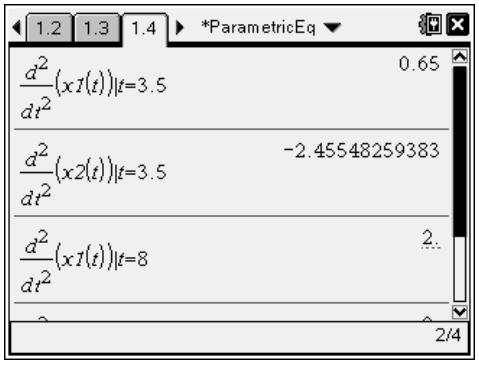
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Gateway Arc Length (TI-Nspire CAS) — 12440*
- *Projectile Motion (TI-Nspire technology) — 10214*
- *Simultaneous Solutions (TI-Nspire technology) — 10092*
- *Warming Up to SimCalc MathWorlds (TI-84 Plus) — 7991*

Part 1 – Parametric Equations – Kinematics

Parametric equations are useful for comparing the motion of various objects. This has application in calculus by finding the *x* and *y* components of the velocity and acceleration. In this activity, two objects are considered to be racing linearly against one another. Have students use the worksheet and answer the questions to investigate page 1.2. Page 1.2 is set up to be modified and used for other equations, including two-dimensional graphs. Press ctrl + G, and change **x1**, **y1** and/or **x2**, **y2**. Students can use the Scratchpad to aid in calculations if necessary. TO access the





Discussion Questions

- What is the acceleration when it is changing direction? *Answer: Although many students will think the acceleration is zero, it is only the instantaneous velocity that is zero. If the acceleration was zero at that instant it would not be changing its velocity.*

When using the CAS (computer algebra system) to find the derivative, a common mistake is to take the derivative with respect to *x* instead of *t*.

Student Solutions

1. The two objects do not start at the same location. They both begin by going backwards, or in the negative *x* direction. At the end, they are both going forward, but the square goes further. In between, they both speed up and slow down, but the diamond's change in velocity is more dramatic. They pass each other once, when the time is about 7.5 units. This could be solved by setting the two equations equal to each other.

2. **a.** The diamond moves faster at 3.5 units.
   b. The square moves faster at 8 units.

3. Let **x1** model the motion of the square and **x2** model the diamond.

$\mathbf{x1}'(t) = 0.15t^2 - 0.4t - 0.5$	$\mathbf{x1}'(3.5) = -0.0625$	$\mathbf{x1}'(8) = 5.9$
$\mathbf{x2}'(t) = 1 - 7\cos(t)$	$\mathbf{x2}'(3.5) = 7.555$	$\mathbf{x2}'(8) = 2.019$

4.

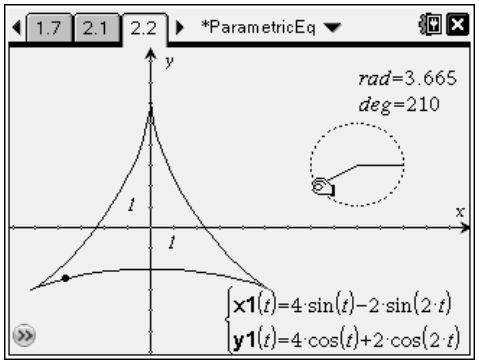
$\mathbf{a1}(t) = \mathbf{x1}''(t) = 0.30t - 0.4$	$\mathbf{x1}''(3.5) = 0.65$	$\mathbf{x1}''(8) = 2$
$\mathbf{a2}(t) = \mathbf{x2}''(t) = 7\cos(t)$	$\mathbf{x2}''(3.5) = -2.455$	$\mathbf{x2}''(8) = 6.926$

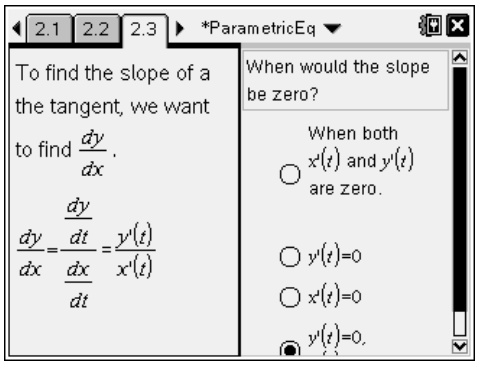
5. At **x1**(3.5), the first derivative is negative and the second derivative is positive. This means it is slowing down as it travels in the negative direction. Similarly, **x2** at $t = 3.5$ has a positive velocity and negative acceleration, indicating it is slowing down as it moves forward.

Part 2 – Parametric Equations – Slope

Students will explore a graph that is not a function but can be graphed with parametric equations. They will grab and move the point on the circle to change the 'time' that corresponds to an angle in this example.

Students are to learn about the vertical and horizontal slope of the tangent.





Student Solutions

6. There is a cusp at 0, $\frac{2\pi}{3}$, and $\frac{4\pi}{3}$ (or at 0°, 60°, and 120°) The derivative is undefined at a cusp. This implies the derivative of **x1**(*t*) is zero at these times.

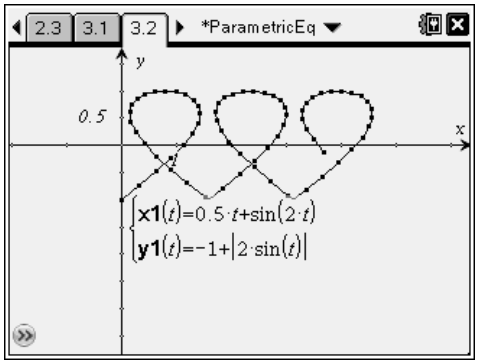
7. A horizontal tangent occurs at (0, 2).

8. This can be found when $y'(t) = 0$, $x'(t) \neq 0$.

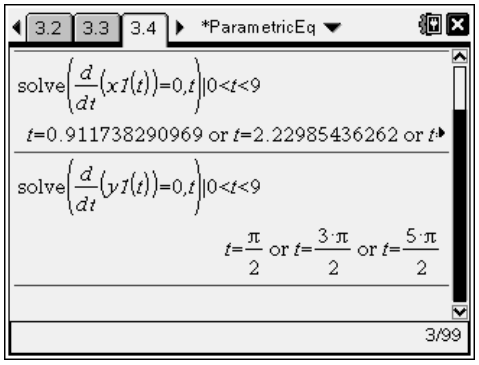
9. $\frac{dy}{dx} = 1$ when $\frac{dy}{dt} = \frac{dx}{dt} \neq 0$
When $t = 4.712388$, $x(t) = -4.000$ and $y(t) = 2.000$.

Part 3 – Homework/Extension

Students will practice and explore interesting graphs on their own and report that they understand the calculus. On page 3.2, students will try to create a set of parametric equations to match the scatter plot. In order to find a solution, students will need to study the slope of the vertical and horizontal tangent lines. You may wish to have students come up with a different set of parametric equations and find the vertical and horizontal tangents of their graph.



The solution to the given problem is shown to the right. On page 3.3 of the solution TI-Nspire document, you will see the values used in the scatter plot. On page 3.4, the answers correspond to the *t*-values of the vertical and horizontal tangents.



Students can also explore a "Parametric Vector Extension." This can be used and modified to explore position *r*, velocity *v*, and acceleration *a* vectors. Click on the minimized slider on the bottom left to change the number of vectors that are show. If you wish to have students explore a different set of parametric equations, press ctrl + G to show the entry line (or hide it later) and change **x1**(*t*) and **y1**(*t*). The velocity and acceleration are calculated in **x2**, **y2**, and **x3**, **y3**.