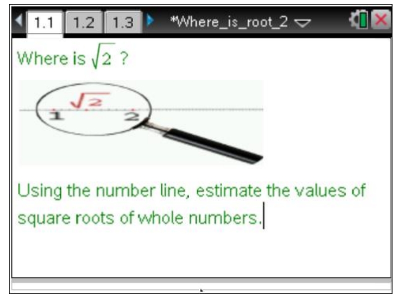

Math Objectives

- Students will use rational numbers to approximate irrational numbers.
- Students will use the number line diagram to estimate the value of square roots.
- Students will use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (CCSS).
- Students will attend to precision (CCSS Mathematical Practice).
- Students will use appropriate tools strategically (CCSS Mathematical Practice).

Vocabulary

- rational and irrational numbers
- decimal approximation
- compound inequality

About the Lesson

- This lesson involves estimating the values of two irrational numbers using a number line with increasing precision of scale.
- As a result, students will:
  - Estimate square roots by 'zooming in' on the values on a number.
  - Find the rational upper and lower bounds for irrational numbers.
  - Explain how to find better and better approximations of an unknown value.

TI-Nspire™ Navigator™ System

- Use Screen Capture to monitor student work and generate class discussions.
- Use Quick Poll to assess students' understanding and generate class discussions.
- Use Live Presenter to project students' screens while they explain their work.

TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Files:
Student Activity
Where_is_Root_2_Student.pdf
Where_is_Root_2_Student.doc

TI-Nspire document
Where_is_Root_2.tns
Where_is_Root_2_Assessment.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

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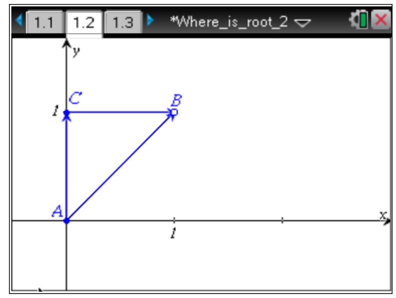


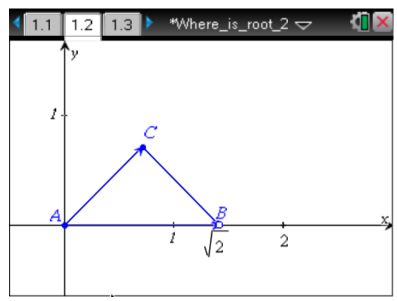
1. Given right isosceles triangle ΔABC. If AC = BC = 1, what is the exact length of AB? Explain how you found this value.

   Answer: Using the Pythagorean theorem, the answer is $AB = \sqrt{AC^2 + BC^2} = \sqrt{2}$.

2. If you were to plot segment AB on the positive side of number line while keeping point A at the origin, what are two consecutive integers between which the point B will land on the number line? Why?

   Answer: The point B will be at the distance $\sqrt{2}$ from the origin because the length of AB remains the same during rotation. Since $\sqrt{1} < \sqrt{2} < \sqrt{4}$, then $\sqrt{2}$ is between 1 and 2.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.



3. Move point B to the x-axis to verify your prediction. What are the lower and upper integer boundaries for the coordinate of the point B? Explain your observations.

   Answer: Point B has an x-coordinate $\sqrt{2}$, and it is located between 1 and 2 on the number line. It is closer to 1 than to 2.

4. How can you make a better approximation for the coordinate of the point B?

   Answer: If we can increase precision of the number line to see smaller tick marks, we can make a better approximation.

Teacher Tip: Encourage students to share their ideas about increasing precision of estimations. Help students to develop the idea of “zooming in” on a point and what this does to the precision of the number line.

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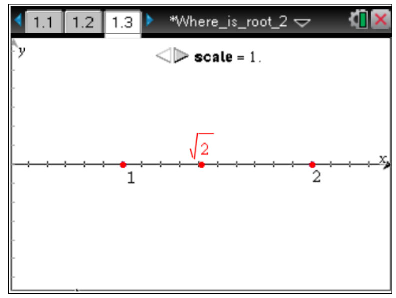


Page 1.3 shows the value of the scale that determines the precision with which you are reading the numbers on the given number line. The value of "scale" is equal to the number of digits after the decimal point you can determine in a number plotted on this number line.

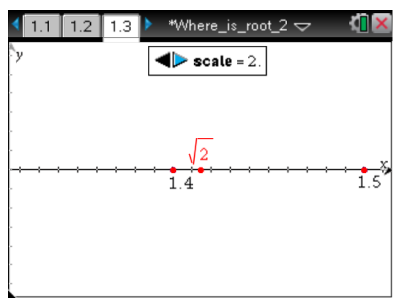
5. When scale = 1, what is the smallest division of the number line? Using the number line, find the closest lower and upper boundaries for $\sqrt{2}$ with the given precision.

   Answer: The smallest division is 0.1. The closest lower and upper boundaries of $\sqrt{2}$ with precision 0.1 are 1.4 and 1.5.

6. Complete the 2nd row of the table below for the answer you found in Question 5, following the example given in the 1st row of the table. Make sure to include:
   - The scale value and the size of smallest division of the number line.
   - The lower and upper boundaries of $\sqrt{2}$ according to the precision of the number line.
   - The compound inequality for $\sqrt{2}$.
   - The distance between the upper and lower boundaries.

   Answer: Shown in the table on the next page under #8.

7. Click the right arrow ▸ to change the scale value to 2. What is the smallest division of the number line in this case? Can you make a better approximation of $\sqrt{2}$ using this number line? Why?



   Answer: This number line has smallest division 0.01. We can make a better approximation because we can determine lower and upper boundaries with the precision of 0.01.

8. Complete the remaining rows in the data table. Click the right arrow ▸ to change the scale.

Answer:

Scale	Smallest division	Lower boundary	Upper boundary	Inequality	Difference between upper and lower boundary
0	1	1	2	$1 < \sqrt{2} < 2$	1
1	0.1	1.4	1.5	$1.4 < \sqrt{2} < 1.5$	0.1
2	0.01	1.41	1.42	$1.41 < \sqrt{2} < 1.42$	0.01
3	0.001	1.414	1.415	$1.414 < \sqrt{2} < 1.415$	0.001
4	0.0001	1.4142	1.4143	$1.4142 < \sqrt{2} < 1.4143$	0.0001
5	0.00001	1.41421	1.41422	$1.41421 < \sqrt{2} < 1.41422$	0.00001
6	0.000001	1.414213	1.414214	$1.414213 < \sqrt{2} < 1.414214$	0.000001

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

Teacher Tip: Help students develop an understanding that they can approximate a number on the number line with precision equal to the precision of the number line. At the same time, precision of the number line is equal to the size of the smallest division (distance between minor tick marks).
By completing the table, they should realize that the precision of approximation is equal to the difference between upper and lower boundary we can find, which is equal to the size of the smallest division of the number line.

9. As precision of the scale increases, what happens to the lower boundary of $\sqrt{2}$? To the upper boundary of $\sqrt{2}$? To the distance between the boundaries? Support your answers.

Answer: As precision increases, the lower boundary of $\sqrt{2}$ increases and the upper boundary of $\sqrt{2}$ decreases. The distance between the boundaries gets smaller and is equal to the smallest division of the number line. This is happening because boundaries get closer and closer to $\sqrt{2}$.

10. Record the best approximation for $\sqrt{2}$ based on your data. What is the precision of your approximation? Why?

    <u>Sample Answers:</u> Student answers will vary. Some students will select the lower boundary, 1.414213, or the upper boundary, 1.414214. Some students will try to estimate the location of $\sqrt{2}$ between these two boundaries, so answers like 1.4142136 are also possible. Explanations should include either the statement of choice of one of boundaries based on precision of the scale or visual observation of how close the point is to one or the other boundaries.

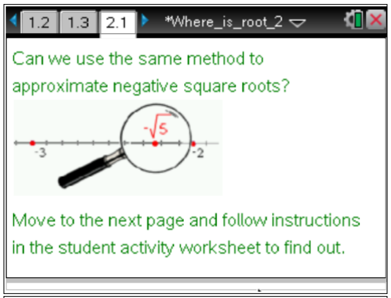
TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

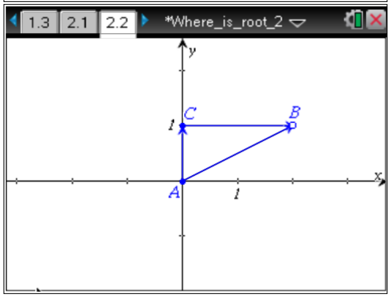
11. Do you observe a pattern in the decimal approximation of $\sqrt{2}$?

    <u>Answer:</u> There is no observed pattern in the decimal approximation of $\sqrt{2}$.

> **Teacher Tip:** This is a good place to discuss with the students that both rational and irrational numbers could have an infinite number of decimals. However, if a number is irrational, there is no pattern to the decimals (e.g., no repeating decimals). With precision of 0.000001 of the scale, we cannot make a statement whether $\sqrt{2}$ will or will not have a pattern.
> The following discussion introduces students to the idea that we can only use proofs to determine whether the number is irrational, since we cannot be sure the pattern does not exist based on approximations only. The number $\sqrt{2}$ has been proven to be irrational. The first proof is attributed to the ancient Greeks, and various proofs have been developed since. Encourage students to use the Internet to learn more about $\sqrt{2}$.

Move to page 2.2.





12. Given right triangle ΔABC, where AC = 1 and BC = 2. What is the exact length of AB? Explain how you found this value.

    Answer: Using the Pythagorean theorem, the answer is $AB = \sqrt{AC^2 + BC^2} = \sqrt{1+4} = \sqrt{5}$.

13. What is the coordinate of the point B if it is plotted on the number line at a distance AB to the right of the origin? What are two consecutive integers between which the point B will land on the number line? Why?

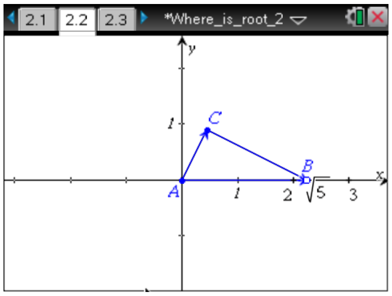
    Answer: Point B will be at the distance $\sqrt{5}$ from the origin because the length of AB remains the same during rotation. Since $\sqrt{4} < \sqrt{5} < \sqrt{9}$, then $\sqrt{5}$ is between 2 and 3.

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

Teacher Tip: in order to explain the lower and upper integer boundaries of $\sqrt{2}$, students will need to compare squares of the numbers. This is an important mathematics method for comparing radicals, that is based on the property that if $a^2 < b^2 < c^2$ for positive numbers, a, b, and c, then $a < b < c$.

14. Move point B to the positive side of the x-axis to verify your prediction. Record the compound inequality for this case.

    Answer: $2 < \sqrt{5} < 3$

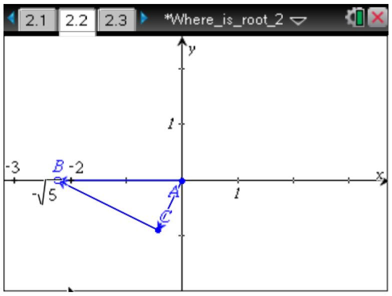


15. What is the coordinate of the point B if it is plotted on the number line at a distance AB to the left of the origin? What are two consecutive integers between which point B will land on the number line? Why?

Answer: The coordinate is $-\sqrt{5}$ because the distance is the same but the point is now plotted in the opposite direction. The point should be between -3 and -2, since -3 is farther from origin than $-\sqrt{5}$ and -2 is closer to the origin than $-\sqrt{5}$.

16. Move point B to the negative side of the x-axis to verify your prediction. Record the compound inequality for this case.

Answer: $-3 < -\sqrt{5} < -2$



> **Teacher Tip:** The negative inequalities are conceptually hard for the students. Reinforce the idea that the numbers to the left are smaller than the numbers to the right on the number line. This should help students to develop correct compound inequalities.

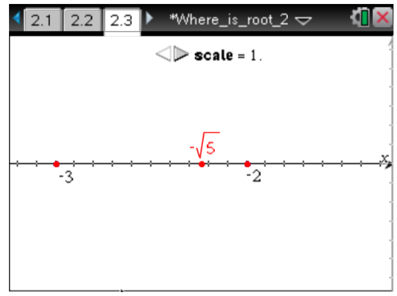
17. What is the same and what is different about the location of point B in these two cases?

Answer: The distance from the origin to point B is the same, $\sqrt{5}$. However, it's x - coordinates are opposite. When point B is on the positive part of the number line, the x -coordinate is $\sqrt{5}$ and when point B is on the negative part of the number line, the x -coordinate is $-\sqrt{5}$.

18. Complete the first row of the data table below. How can you make a better approximation for the coordinate of the point B?

Answer: If we can increase the precision of the number line to see smaller tick marks, we can make better approximations.

Move to page 2.3.



19. Complete the table as you increase the scale value. Click the right arrow ▸ to change the scale.

Answer:

Scale	Smallest division	Lower boundary	Upper boundary	Inequality	Difference between upper and lower boundary
0	1	-3	-2	$-3 < -\sqrt{5} < -2$	1
1	0.1	-2.3	-2.2	$-2.3 < -\sqrt{5} < -2.2$	0.1
2	0.01	-2.24	-2.23	$-2.24 < -\sqrt{5} < -2.23$	0.01
3	0.001	-2.237	-2.236	$-2.237 < -\sqrt{5} < -2.236$	0.001
4	0.0001	-2.2361	-2.2360	$-2.2361 < -\sqrt{5} < -2.2360$	0.0001
5	0.00001	-2.23607	2.23606	$-2.23607 < -\sqrt{5} < -2.23606$	0.00001
6	0.000001	-2.236067	-2.236068	$-2.236067 < -\sqrt{5} < -2.236068$	0.000001

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

20. As precision of the scale increases, what happens to the lower boundary of $-\sqrt{5}$? To the upper boundary of $-\sqrt{5}$? To the distance between the boundaries? Support your answers.

Answer: As precision increases, the lower boundary of $-\sqrt{5}$ increases and the upper boundary of $-\sqrt{5}$ decreases. The distance between the boundaries gets smaller and is equal to the smallest division of the number line. This is happening because boundaries get closer and closer to $-\sqrt{5}$.

21. Record the best approximation for $-\sqrt{5}$ based on your data. What is the precision of your approximation? Why?

    Sample Answers: Student answers will vary. Some students will select the lower boundary, -2.236067, or the upper boundary, -2.236068. Since the value of $-\sqrt{5}$ is so close to -2.236068, students are most likely pick this value. Explanations should include the statement of choice of one of boundaries based on precision of the scale and visual observation of how close the point is to one or the other boundaries.

> **TI-Nspire Navigator Opportunity: *Quick Poll***
> **See Note 1 at the end of this lesson.**

22. Do you observe a pattern in the decimal approximation of $-\sqrt{5}$?

    Answer: There is no observed pattern in the decimal approximation of $-\sqrt{5}$.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- Using the number line to approximate irrational numbers with different precision.
- Writing compound inequalities showing irrational numbers bounded and approximated by rational numbers.
- Developing methods for approximating and comparing irrational numbers.

Assessment

Use the TI-Nspire document Where_is_root_2_assessment.tns to assess student understanding of the material. You can send the file at the end of the lesson or use questions one at a time and send them to the students as Quick Polls.

TI-Nspire Navigator

Note 1

Name of Feature: Quick Poll

Collect student answers to questions using *Quick Poll*.

Note 2

Name of Feature: Live Presenter

Use *Live Presenter* to let students demonstrate and explain how they determine the more precise rational approximations for the irrational numbers using the number line.