Addition and Subtractions of Rational Numbers—Part 2

MATH NSPIRED

TEACHER NOTES

Math Objectives

- Students will represent signed mixed numbers and the results of the addition and subtraction of signed mixed numbers on a number line.
- Students will understand subtraction of mixed numbers as adding the additive inverse.
- Students will apply and extend previous understandings of addition and subtraction to addition and subtraction of rational numbers; represent addition and subtraction on a horizontal or vertical number line (CCSS).
- Students will model with mathematics (CCSS Mathematical Practice).

Vocabulary

- rational number
- equivalent fractions
- mixed numbers

About the Lesson

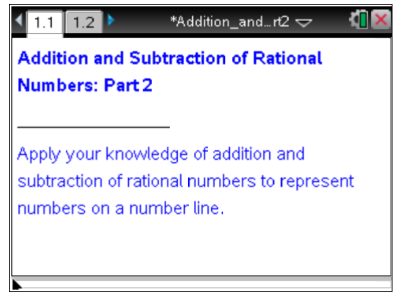
- This lesson involves representing addition and subtraction of signed mixed numbers on a number line for a randomly generated target sum or difference.
- As a result, students will:
  - Represent addition and subtraction of signed mixed numbers on a number line.
  - Calculate sums and differences, and use the TI-Nspire to check the accuracy of their calculations.

TI-Nspire™ Navigator™

- Send file to students.
- Use Class Capture to examine patterns that emerge.
- Use Live Presenter to debrief solutions.
- Use Quick Poll to assess students' understanding.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.

Lesson Files:

Student Activity

- Add_Sub_Rational_Numbers_Part2_Student.pdf
- Add_Sub_Rational_Numbers_Part2_Student.doc
- Add_Sub_Rational_Numbers_Part2_Supplement.doc

TI-Nspire document

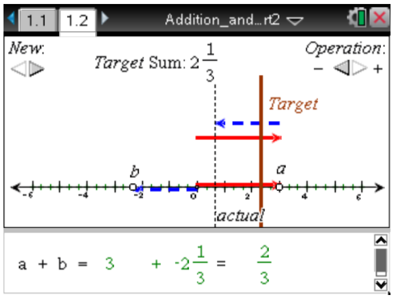
- Add_Sub_Rational_Numbers_Part2.tns

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand ready to grab the point. Then press ctrl to grab the point and close the hand.

Teacher Tip: The two parts of this activity can be used as a single lesson or independently at different times.

Move to page 1.2.



Page 1.2 shows a number line with two movable points:

- a represented by a solid vector, and
- b represented by a dotted vector.

The sum (or difference) of a and b is modeled using vectors above the number line:

- The **red** vector pointing to the right will always represent a positive number.
- The **blue** vector pointing to the left will always represent a negative number.
- The solid vertical line that goes through the corresponding point on the number line represents a target sum or difference (T).
- As you manipulate the values of a and b, the actual value of the sum (or difference) is represented by a vertical dotted line that goes through the corresponding point on the number line.

Tech Tip: Students can select a new target value by clicking on the left or right arrows below the label *New*. The target values are generated randomly. Students can change the operation from addition to subtraction by clicking on the *Operation* arrows — left arrow for subtraction and right arrow for addition. After using either of the arrows, students should press esc in order to be able to grab and drag points.

Tech Tip: Students can select a new target value by tapping on the left or right arrows below the label *New*. The target values are generated randomly. Students can change the operation from addition to subtraction by tapping on the *Operation* arrows — left arrow for subtraction and right arrow for addition. To drag the points for *a* or *b*, students should touch the point and drag their finger while still touching the iPad.

Teacher Tip: The points a and b can be grabbed and dragged along the number line while the values of these numbers will be displayed at the bottom of the screen in the equation $a+b=$. Negative numbers are represented with a blue vector pointing to the left. Positive numbers are represented with a red vector pointing to the right. The sum (or difference) of two vectors is modeled above the number line. When students find values for a and b so that the actual value of the sum (or difference) is equal to the target value, the vertical dotted line will match the vertical solid line, and a check mark will appear to the right of the equation at the bottom of the screen.

Teacher Tip: All students will have the same problem and information when Page 1.2 is first opened. The teacher can use this problem to model how to use this number line investigation or to have a whole class discussion of the approaches used by the students after they have explored the problem.

1. For each case listed in the table on the next page, you are given a value for point a and a value for T. Using the interactive number line model on Page 1.2, grab and drag point a to the given value on the number line.

2. Then, calculate the value of b, so that the sum (or difference) of a and b is equal to the target value T. Record your answers and calculations in the provided table on the next page:
   a. Compute b, and show your work.
   b. Draw a number line representation of the problem with given values of a and T, and mark the value of b on the number line.
   c. Write an inequality that represents relationships between values a, b, T, and zero.

Teacher Tip: Students can choose different strategies to find the value of b. For instance, in case 1) the first example given in the table below demonstrates the strategy of adding up to the nearest integer, and then adding more to achieve the final sum. The second example demonstrates the use of equivalent fractions to rewrite the sum, followed by the traditional approach, if $T=a+b$, then $b=T-a$.

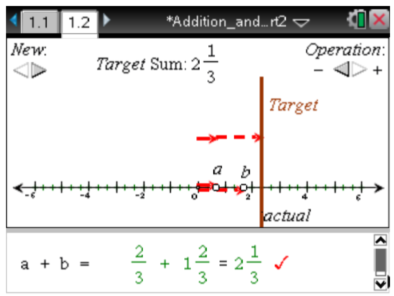
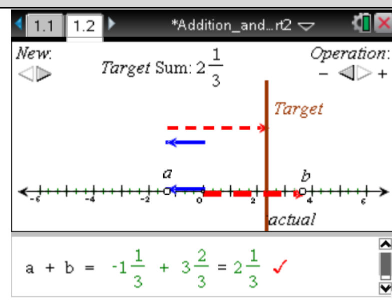
In case 2) the first example demonstrates the use of the fact that the number and its additive inverse add up to zero The second example uses the traditional approach, if $T=a+b$, then $b=T-a$.

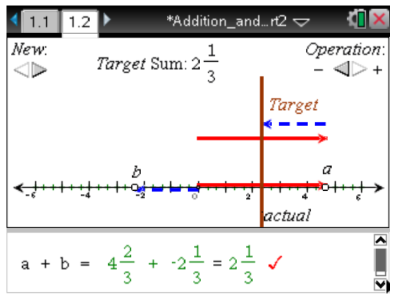
In case 4) the first example demonstrates the property of subtraction of rational numbers as addition of the additive inverse, $p - q = p + (-q)$. The second example uses the traditional approach, if $T = a - b$, then $b = a - T$. Encourage discussion of different strategies that can be used to solve addition and subtraction problems with the students.

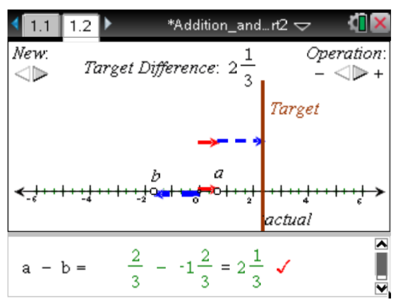
3. Check your calculations by moving point b to the location on the number line that you found. Does the *actual* sum equal the target sum (does the vertical dotted line match the vertical solid line)? What does it mean that the actual sum is equal to the target sum?

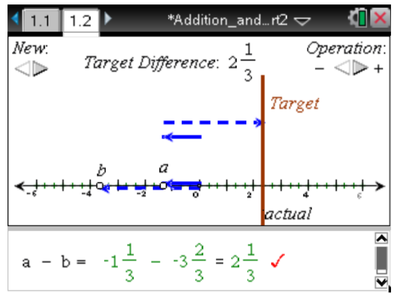
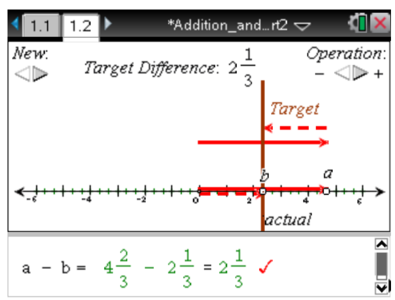
Sample Answers: When the actual sum is equal to the target sum, the dotted vertical line will coincide with the solid vertical line, and the sum of the two vectors will end on the target line.

Teacher Tip: Facilitate discussion with the students after they use the number line to check their calculations. Ask students to provide visual descriptions of the actual sum in relation to the target sum. Encourage students to talk about different representations of the actual sum on the handheld screen, e.g., as a point on the number line, as a dotted vertical line through this point, and as a sum of two vectors, a and b. Help students to make a connection between addition of numbers and movements along the number line according to vectors representing these numbers. Ask them whether their predictions about properties of b were confirmed or not. Ask students to explain what could have caused their mistakes and how they can avoid them in the future. In their explanations, students should demonstrate understanding of $T = a + b$ as the number located a distance $|b|$ from a, in the positive or negative direction depending on whether b is positive or negative. Using vector models encourages students to talk about the length of vector b that represents the $|b|$ and the direction of vector b that represents the sign of the number, positive when directed to the right and negative when directed to the left.

	Case	I. Number line representation.	II. Calculations.	III. Inequality
1)	$T = a + b = 2\frac{1}{3}$ $a = \frac{2}{3}$		$\frac{2}{3} + \frac{1}{3} = 1, \quad 1 + 1\frac{1}{3} = 2\frac{1}{3}$ $\therefore b = \frac{1}{3} + 1\frac{1}{3} = 1\frac{2}{3}$ OR $2\frac{1}{3} = 1\frac{4}{3}$ $+ \quad -\frac{2}{3} = -\frac{2}{3}$ $1\frac{2}{3}$	$0 < a < b < T$
	Case	**I. Number line representation.**	**II. Calculations.**	**III. Inequality**
2)	$T = a + b = 2\frac{1}{3}$ $a = -1\frac{1}{3}$		$-1\frac{1}{3} + 1\frac{1}{3} = 0, \quad 0 + 2\frac{1}{3} = 2\frac{1}{3}$ $\therefore 1\frac{1}{3} + 2\frac{1}{3} = 3\frac{2}{3}$ OR $b = T - a = 2\frac{1}{3} - \left(-1\frac{1}{3}\right) = 2\frac{1}{3} + 1\frac{1}{3} = 3\frac{2}{3}$	$a < 0 < T < b$

	Case	I. Number line representation.	II. Calculations.	III. Inequality
3)	$T=a+b=2\frac{1}{3}$ $a=4\frac{2}{3}$		$b=T-a=2\frac{1}{3}-\left(4\frac{2}{3}\right)=-2\frac{1}{3}$ OR $a-T=4\frac{2}{3}-1\frac{1}{3}=2\frac{1}{3}$ $\therefore b=T-a=-2\frac{1}{3}$	$b<0<T<a$

	Case	I. Number line representation.	II. Calculations.	III. Inequality
4)	$T=a-b=2\frac{1}{3}$ $a=\frac{2}{3}$		$\frac{2}{3}-\left(-\frac{1}{3}\right)=1,\quad 1-\left(-1\frac{1}{3}\right)=2\frac{1}{3}$ $\therefore b=\left(-\frac{1}{3}\right)+\left(-1\frac{1}{3}\right)=-1\frac{2}{3}$ OR $b=a-T=\frac{2}{3}-\left(2\frac{1}{3}\right)=-1\frac{2}{3}$	$b<0<a<T$

	Case	I. Number line representation.	II. Calculations.	III. Inequality
5)	$T=a-b=2\frac{1}{3}$ $a=-1\frac{1}{3}$		$-1\frac{1}{3}-\left(-1\frac{1}{3}\right)=0,\quad 0-\left(-2\frac{1}{3}\right)=2\frac{1}{3}$ $\therefore b=-1\frac{1}{3}+-2\frac{1}{3}=-3\frac{2}{3}$ OR $b=a-T=-1\frac{1}{3}-2\frac{1}{3}=-3\frac{2}{3}$	$b<a<0<T$
	Case	**I. Number line representation.**	**II. Calculations.**	**III. Inequality**
6)	$T=a-b=2\frac{1}{3}$ $a=4\frac{2}{3}$		$4\frac{2}{3}-1\frac{2}{3}=3,\quad 3-\frac{2}{3}=2\frac{1}{3}$ OR $\therefore b=1\frac{2}{3}+\frac{2}{3}=2\frac{1}{3}$ $b=a-T=4\frac{2}{3}-2\frac{1}{3}=2\frac{1}{3}$	$0<b=T<a$

TI-Nspire Navigator Opportunity: ***Quick Poll, Class Capture, and Live Presenter***
See Note 1 at the end of this lesson.

Teacher Tip: The next two questions involve addition and subtraction problems with $T < 0$. Students could work in pairs in a game format using the *Did You Hit the Target? Rules and Scoring Sheet* supplied in the Add_Sub_Rational_Numbers_Part2_Supplement.pdf. Students should follow instructions below to generate problems for the game.

4. On Page 1.2, select *Operation* to select addition. Select *New* until the Target value is negative. Be patient. This might take several selections. Record this value $T =$ _________ .

   a. Choose the value of a so that $a < T$. Record this value, $a =$ _________ . Calculate the value of b, and locate its position on the number line.

   b. Choose the value of a so that $T < a < 0$. Record this value, $a =$ _________ . Calculate the value of b, and locate its position on the number line.

   c. Choose the value of a so that $a > 0$. Record this value, $a =$ _________ . Calculate the value of b, and locate its position on the number line.

Answer: $T = a + b; \quad T < 0$

a. $T = a + b = -4\frac{5}{6}$, $a = -5\frac{1}{3}, b = \frac{1}{2}$, $a < T < 0 < b$

b. $T = a + b = -4\frac{5}{6}$, $a = -2\frac{1}{3}, b = -2\frac{1}{2}$, $T < b < a < 0$

c. $T = a + b = -4\frac{5}{6}$, $a = \frac{5}{6}, b = -5\frac{2}{3}$, $b < T < 0 < a$

5. Select *Operation* to select subtraction. Select *New* until the Target value is negative. Be patient. This might take several selections. Record this value $T =$ _________ .

   a. Choose the value of a so that $a < T$. Record this value, $a =$ _________ . Calculate the value of b, and locate its position on the number line.

   b. Choose the value of a so that $T < a < 0$. Record this value, $a =$ _________ . Calculate the value of b, and locate its position on the number line.

   c. Choose the value of a so that $a > 0$. Record this value, $a =$ _________ . Calculate the value of b, and locate its position on the number line.

   Answer: $T = a - b$; $T < 0$

   a. $T = a - b = -4\frac{5}{6}$, $a = -5\frac{1}{3}, b = -\frac{1}{2}$, $a < T < b < 0$

   b. $T = a - b = -4\frac{5}{6}$, $a = -2\frac{1}{3}, b = 2\frac{1}{2}$, $T < a < 0 < b$

   c. $T = a - b = -4\frac{5}{6}$, $a = \frac{5}{6}, b = 5\frac{2}{3}$, $T < 0 < a < b$

6. Refer to the number line on Page 1.2 to complete the **Addition and Subtraction of Mixed Numbers – Summary Chart**, which is located in Add_Sub_ Rational_Numbers_Part_2_Supplement.pdf. Explain your results for each case in the chart.

   Answer: See **Addition and Subtraction of Mixed Numbers – Completed Chart** in the Add_Sub_Rational_Numbers_Part_2_Supplement.pdf document.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

Teacher Tip: Ask students to demonstrate these cases using the number line on Page 1.2. Consider different examples and help students to generalize their findings about the results of addition and subtraction of signed rational numbers. Encourage students to use vector models to provide explanations. Make connections between vectors and actual movement, (for example, adding a positive and a negative number can be modeled as going to the right and then going to the left).

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to use equivalent fractions and common denominators for the addition and subtraction of signed mixed numbers.
- How to represent the location of signed mixed numbers on a number line.
- How to predict and find the location of numbers resulting from addition and subtraction of signed mixed numbers on the number line.
- How to generalize the results of addition and subtraction of signed mixed numbers.

Assessment

Generate new problems using TI-Nspire documents. Use questions provided in the lesson for these problems to assess students' understanding of addition and subtraction of signed mixed numbers.

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Note 1

Quick Poll, Class Capture, and Live Presenter

Use Quick Poll to evaluate students understanding of equivalent fractions, using common denominators, adding up to an integer and additive inverses; use Class Capture to sort the solutions of each group; and use Live Presenter to allow each group to debrief their solutions.

Note 2

Live Presenter

Use Live Presenter for students to provide examples for each case using the interactive number line. Encourage students explain their findings and generalize results of addition and subtraction of signed numbers.