

Circular Probability

Student Activity

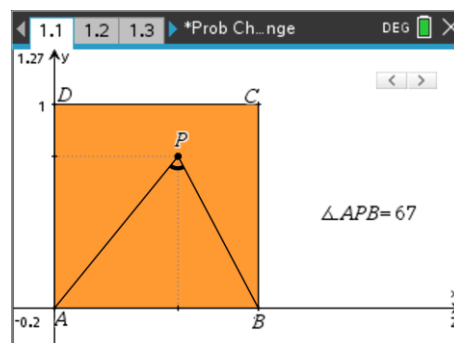
7 8 9 10 11 12



Problem to Solve

A point P is placed inside a unit square: ABCD. If point P is able to move around randomly inside the square, what is the probability that the angle: $\angle APB \geq 90^\circ$.

The diagram opposite shows a sample point where $\angle APB = 67^\circ$. The slider can be used to generate points to gain an insight to the problem. Open the Prob Challenge document to explore this probability.



Question: 1.

Click on the slider 10 times and record the quantity of successes ($\angle APB \geq 90^\circ$) and failures ($\angle APB < 90^\circ$). Use this data to provide an estimate for the probability.

Building a Simulation

A Spreadsheet application has been inserted in Page 1.2 of the TI-Nspire "Prob-Challenge" document.

Navigate to the formula bar in the first column (data) and generate an automatic data capture of the angle (APB) on Page 1.1.

[menu] > Data > Data Capture > Automatic

From the variable menu [var], select variable APB or type: APB

Each time variable APB is changed on Page 1.1, the results will be recorded automatically.

The next step is to calculate the estimated probability based on the 'data'.

In cell B1 type the formula:

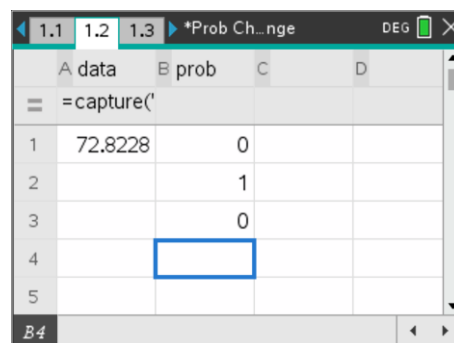
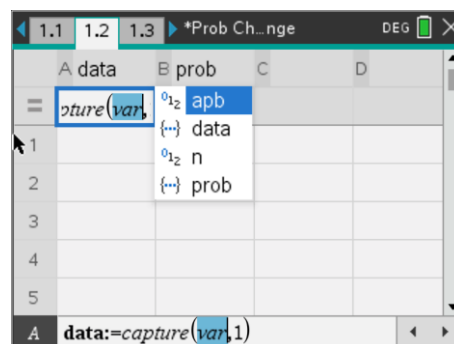
$$=countif(data, ? \geq 90)$$

In cell B2 type the formula:

$$=dim(data)$$

In cell B3 type the formula:

$$= \frac{b1}{b2}$$



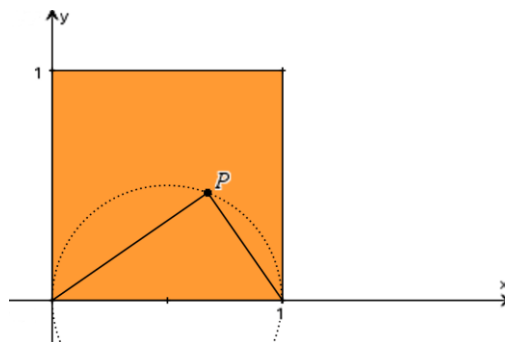
The formula in cell B1 will count how many times the angle in the data column is greater than (or equal) to 90° . The formula in cell B2 counts how many data points have been collected (dimension of the data list) and cell B3 calculates the proportion of successes from the total number of trials. To see an approximate value for this result the formula can be changed to: $=approx(b1/b2)$.

Question: 2.

Click on the slider repeatedly (or animate) to collect at least 100 data points. Check the spreadsheet and record the estimate for the probability of success.

Theory

The theoretical probability is related to Thales theorem. A circle has been drawn with centre at (0.5, 0) passing through (0, 0) and (1, 0). Point P is on the circumference of the circle.

**Question: 3.**

What is the angle, according to Thales theorem at point P?

Question: 4.

Use your answer to Question 3 to determine the proportion of the area enclosed by the semi-circle within the unit square; and hence the probability that for the randomly generated point P is such that: $\angle APB \geq 90^\circ$.

Question: 5.

Compare the exact answer (Question 4) and the approximate one obtained using the simulation.

Extension

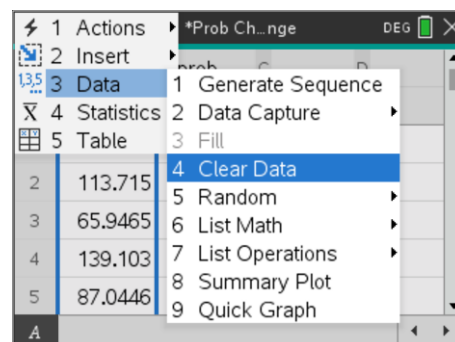
In this part of the investigation, $\angle APB$ can be an angle other than 90° . Start by exploring specific angles (see below) and work towards a general formula for the probability for an angle x such that $\angle APB \geq x$. In each case a probability estimate can be obtained by changing the corresponding spreadsheet formula. The data can be cleared out if desired.

To clear the existing data, navigate to the top of the data column (Column A) and keep on arrowing upwards until the column is highlighted.

[menu] > Data > Clear Data

This will clear data without deleting the automatic data capture command in the formula bar.

Change the formula in cell B1 then return to Page 1.1 to generate data.



Explore probabilities for each of the following:

Angle:	Simulation	Theoretical:
45°		
60°		
120°		

Determine the general probability for an angle x° . [Hint: Consider all scenarios]