Crossing the Asymptote

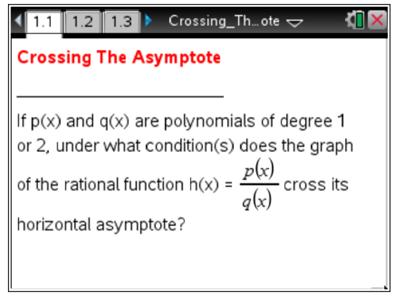
Student Activity

Name ________________

Class ________________

Open the TI-Nspire document *Crossing_The_Asymptote.tns.*

In this activity, you will explore the following question: If $p(x)$ and $q(x)$ are polynomials of degree 1 or 2, under what condition(s) does the graph of the rational function $r(x)=\frac{p(x)}{q(x)}$ cross its horizontal asymptote?



Recall that a rational function $r(x)=\frac{p(x)}{q(x)}$ is the quotient of two polynomials. When the degree of the numerator is less than or equal to the degree of the denominator, a **horizontal asymptote** might exist. If the degree of the numerator equals the degree of the denominator, a horizontal asymptote exists at y = (ratio of leading coefficients of numerator and denominator); and if the degree of the numerator is less than that of the denominator, a horizontal asymptote exists at $y=0$. For example, the horizontal asymptote (if it exists) of $r(x)=\frac{c*x^2+d*x+e}{g*x^2+h*x+k}$ is $y=\frac{c}{g}$, and the horizontal asymptote (if it exists) of $r(x)=\frac{c*x+d}{e*x^2+g*x+h}$ is $y=0$.

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1. Consider an example where when both $p(x)$ and $q(x)$ are linear:

$$p(x)=a\cdot x+21.3,\quad q(x)=2\cdot x+b \text{ where } a\neq 0.$$

   Set the value of the slider **b** to –6. Then scroll through the values of slider **a** from –6 to 6, and make a note of the cases when the graph of $y=f1(x)$ crosses its asymptote $y=f2(x)$. Ignore $a=0$ since we are only considering values of $a\neq 0$. Set **b** to –5, and scroll through the values of **a** noting any cases where the graph crosses its asymptote. Repeat this process for values of **b** from –6 to 6. What pairs of values (if any) for **a** and **b** did you note?

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2. In general, make a conjecture about the sets of values (if any) of $\{c,d,e,f\}$ where the graph of the rational function $f3(x)=\frac{c*x+d}{e*x+f}$ crosses its horizontal asymptote $f4(x)=\frac{c}{e}$. [Assume $c\neq 0, e\neq 0$, and $e\cdot x+f$ is not a multiple of $c\cdot x+d$.]

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3. Test your conjecture. The functions $f3(x)$ and $f4(x)$ have been defined. Enter $solve(f3(x)=f4(x),x)$. Does this result validate your conjecture? Why or why not? If your conjecture was not correct, revise it so the resulting statement is correct.

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4. Consider an example where when p(x) is linear and q(x)is quadratic:

$$p(x)=a\cdot x+1.3\ ;\ q(x)=2\cdot x^2+b\cdot x+3.1 \text{ where } a\neq 0.$$

Set the value of the slider **b** to –6. Then scroll through the values of slider **a** from –6 to 6, and make a note of the cases when the graph of $y=f1(x)$ crosses its asymptote, $y=f2(x)$. Ignore **a** = 0 since we are only considering values of $a\neq 0$. Then set **b** to –5, and scroll through the values of noting any cases where the graph crosses its asymptote. Repeat this process for values of b from –6 to 6. Describe the pairs of values of **a** and **b** that you noted.

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5. In general, make a conjecture about the sets of values $\{c,d,e,g,h\}$ where the graph of the rational function $f3(x)=\frac{c*x+d}{e*x^2+g*x+h}$ does **not** cross its horizontal asymptote $f4(x)=0$. [Assume $c\neq 0, e\neq 0$, and $e\cdot x^2+g\cdot x+h$ is not a multiple of $c\cdot x+d$]

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6. Test your conjecture. The functions $f3(x)$ and $f4(x)$ have been defined. Enter $solve\left(f3(x)=f4(x),x\right)$. Does this result validate your conjecture? Why or why not? If your conjecture was not correct, revise it so the resulting statement is correct.

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7. Consider an example where when both p(x) and q(x) are quadratic:

$$p(x)=a\cdot x^2+6\cdot x+1.3;\; q(x)=2\cdot x^2+b\cdot x-3.1 \text{ where } a\neq 0.$$

a. Set the value of the slider **b** to –6. Then scroll through the values of slider **a** from –6 to 6, and enter the value of ***a*** (if one exists) when the graph of $y=f1(x)$ does **not** cross its asymptote, $y=f2(x)$, in the table. Ignore **a** = 0 since we are only considering values of $a\neq 0$. Then set **b** to –5, and scroll through the values of **a.** Repeat this process for values of **b** from –6 to 6.

Hint: For a given value of ***b***, there is at most one value of ***a*** for which the graph does not cross its asymptote.

b	–6	–5	–4	–3	–2	–1	1	2	3	4	5	6
a												

b. The boxes below –1 and 1 are blank. If the values of the sliders for **a** and **b** were not limited, what values would go in each of these two boxes?

c. Make a conjecture about the relationship between ***a*** and ***b*** that is true for the rational functions in this set whose graph does **not** cross its horizontal asymptote.

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8. In general, make a conjecture about the relationship between $\{c,d,g,h\}$ if the graph of the rational function $f3(x)=\dfrac{c*x^2+d*x+e}{g*x^2+h*x+k}$ does **not** cross its horizontal asymptote $f4(x)=\dfrac{c}{g}$. [Assume $c\neq 0, g\neq 0$ and $g\cdot x^2+h\cdot x+k$ and $c\cdot x^2+d\cdot x+e$ do not have a common linear factor.]

Move to page 3.3.

9. Test your conjecture. The functions $f3(x)$ and $f4(x)$ have been defined. Enter $solve\left(f3(x)=f4(x),x\right)$. Does this result validate your conjecture? Why or why not? If your conjecture was not correct, revise it so the resulting statement is correct.