Math Objectives

- Students will explore some regular polygons to see which of these will tessellate in the plane. Students will use their observations to answer the question, "Which regular polygons will tessellate in the plane and why?"

Vocabulary

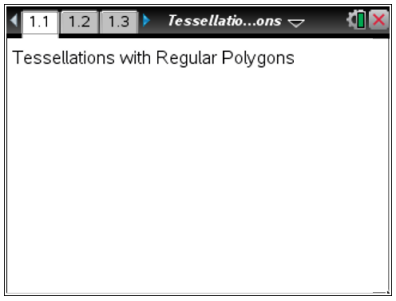
- regular polygon
- pentagon, hexagon, heptagon, octagon

About the Lesson

- The estimated time for this activity is 30 to 45 minutes.
- Send the file.tns to student handheld devices. If you are planning for students to create the file, take time to follow the directions prior to facilitating the process with students.
- This activity is designed to be student-centered, with the teacher acting as a facilitator while students work cooperatively. The student worksheet is intended to guide students through the activity and provide a place to record their answers.

TI-Nspire™ Navigator™ System

- Use Class Capture to observe students' work as they proceed through the activity.
- Use Live Presenter to have a student illustrate how he/she used a certain tool.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.

Lesson Files:

Create Instructions

Tessellations_with_Regular_Polygons_Create.pdf

Student Activity

Tessellations_with_Regular_Polygons_Student.pdf

Tessellations_with_Regular_Polygons_Student.doc

TI-Nspire document

Tessellations_with_Regular_Polygons.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

Discussion Points and Possible Answers

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the arrow until it becomes a hand (☜). Press ctrl [ ] to grab the point and close the hand (☜).

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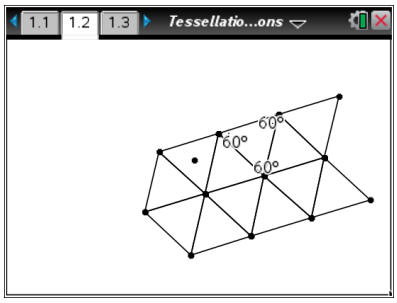

Part 1 – Tessellating a regular polygon

Definitions: A tessellation is a covering, or tiling, of a plane with a pattern of figures so there are no overlaps or gaps. A monohedral tessellation is a tessellation made up of congruent copies of one figure.

A regular polygon is a polygon with equal sides and equal interior angles. An equilateral triangle and a square are both examples of regular polygons.

The equilateral triangle in the center has been tessellated in the plane.

1. What is the measure of each interior angle of the triangle?

   Answer: 60°

2. How many angles come together at one vertex of the tessellation?

   Answer: 6

3. What is the sum of the measures of the angles at one vertex of tessellation?

   Answer: 360°

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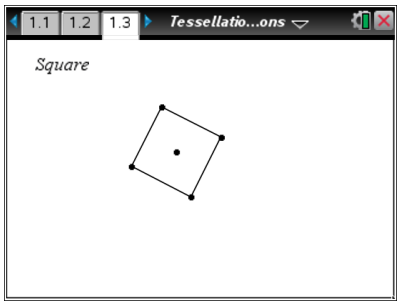


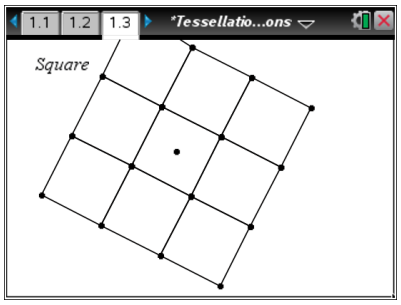
4. What type of regular polygon is shown?

   Answer: a square

5. What is the measure of each interior angle of the polygon?

Answer: 90°

> **Teacher Tip:** To measure an angle, press **Menu > Measurement > Angle**. Click three times in the order "point-vertex-point" as when naming an angle. The vertex of the angle to be measured must be the second point selected. Be sure students click on existing points to avoid extra confusing points appearing. Press esc to exit the **Measurement** tool.



Step 1: Press **Menu > Transformation > Reflection** to select the **Reflection** tool from the Transformation menu.

Step 2: Move the pointer to a side (not a vertex) of the square, and press ctrl twice. (The first click selects the entire regular polygon as the pre-image for the reflection. The second click selects the side of the polygon as the line of reflection.)

Step 3: The **Reflection** tool is still active. Continue clicking twice on any side of the original square or on any side of a reflected image.

Step 4: Press esc to exit the **Reflection** tool.

6. Does the square tessellate?

Answer: yes

7. How many angles come together at one vertex of the tessellation?

Answer: 4

8. What is the sum of the measures of the angles at one vertex of tessellation?

Answer: 360°

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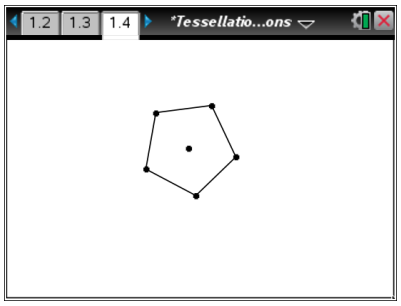


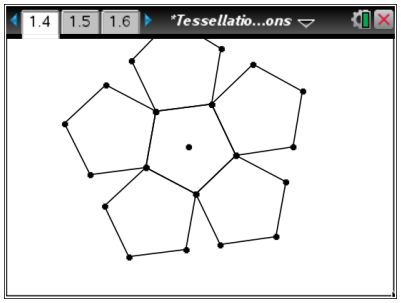
9. What type of regular polygon is shown?

Answer: a pentagon

10. What is the measure of each interior angle of the polygon?

    Answer: 108°

11. Repeat Steps 1–4 above to help you answer the question, "Does the polygon tessellate?"



    Answer: No, the sides do not join and there is extra space or a gap.

12. How many angles come together at one vertex?

    Answer: Three angles from the pentagons come together but they do not tile.

13. What is the sum of the measures of the angles at one vertex?

    Answer: 3 × 108° = 324°

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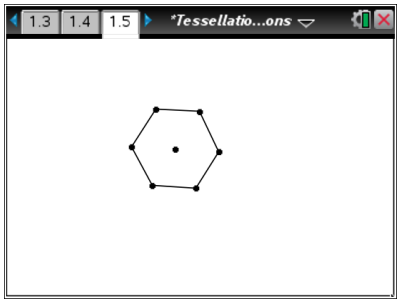


14. What type of regular polygon is shown?

    Answer: a hexagon

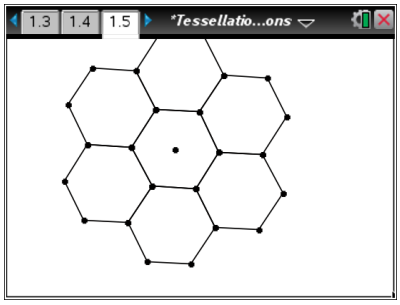
15. What is the measure of each interior angle of the polygon?

    Answer: 120°

16. Do you think that the polygon will *tessellate*? Why or why not?

    Answer: Yes, because each interior angle has a measure of 120° and 120 is a factor of 360. I think that the sum of the measures of the angles at a tessellation vertex will be 360°.

17. Repeat Steps 1–4 to test your conjecture in question 16.



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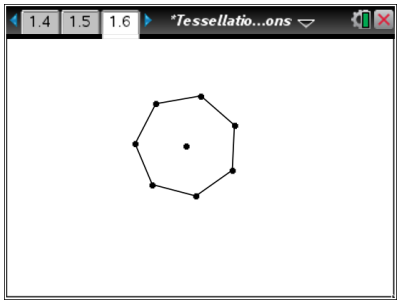


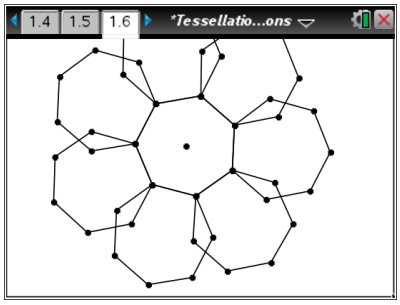
18. What type of regular polygon is shown?

Answer: a heptagon

19. Do you think that the polygon will *tessellate*? Why or why not?

Answer: No, because each interior angle has a measure of ≈128.6°and that is not a factor of 360. I think that the sum of the measures of the angles at a tessellation vertex will not be 360°.

20. Repeat Steps 1–4 to test your conjecture in question 19.



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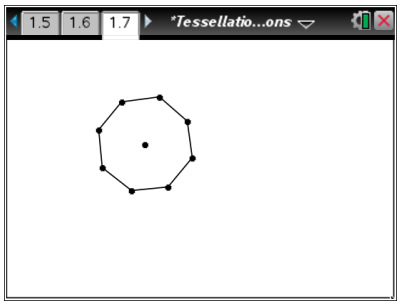


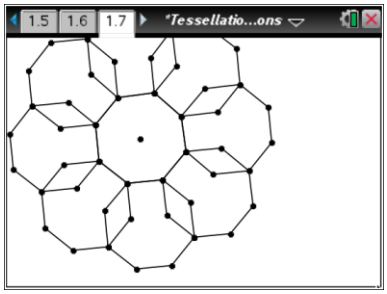
21. What type of regular polygon is shown?

Answer: an octagon

22. Do you think that the polygon will *tessellate*? Why or why not?

Answer: No, because each interior angle has a measure of 135° and 135 is not a factor of 360. I think that the sum of the measures of the angles at a tessellation vertex will not be 360°.

23. Repeat Steps 1–4 to test your conjecture in question 22.



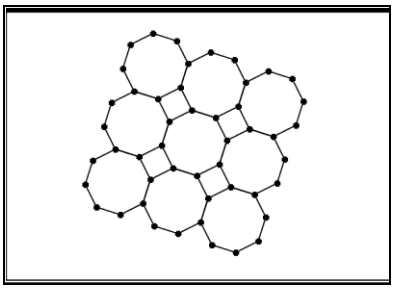
24. Use the table below to organize the information that you have observed. The table can be used to help you generalize and make a conjecture about *monohedral tessellations* of regular polygons.

Regular Polygon	Measure of Interior Angle	Does the polygon tessellate?	If the polygon tessellates, what is the sum of the measures at a tessellation vertex?
Triangle	**60°**	**Yes**	**360°**
Square	**90°**	**Yes**	**360°**
Pentagon	**108°**	**No**	**—**
Hexagon	**120°**	**Yes**	**360°**
Heptagon	**128.6°**	**No**	**—**
Octagon	**135°**	**No**	**—**

25. For the polygons that tessellate, what do you observe about the sum measures of the angles at a tessellation vertex? State this observation as a conjecture.

Answer: The sum of the measures of the angles at a tessellation vertex is 360°.
If the measure of an interior angle of a regular polygon is a factor of 360, then the polygon will tessellate and the sum of the measures of the angles at a tessellation vertex will be 360°.

Part 2 – Dihedral tessellations



26. The figure at the right shows a **dihedral tessellation**—a tessellation using congruent copies of two different shapes.

    a. What two shapes are used for the tessellation shown?

    Answer: a square and a regular octagon

b. Does this new tessellation follow the same rules you discovered in the first part of the activity for monohedral tessellations? In what way?

Answer: Yes, because the sum of the measures of the angles at a tessellation vertex is 135° + 135° + 90° = 360°.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- Which of the given regular polygons will tessellate in the plane.
- Why these regular polygons do or do not tessellate in the plane.