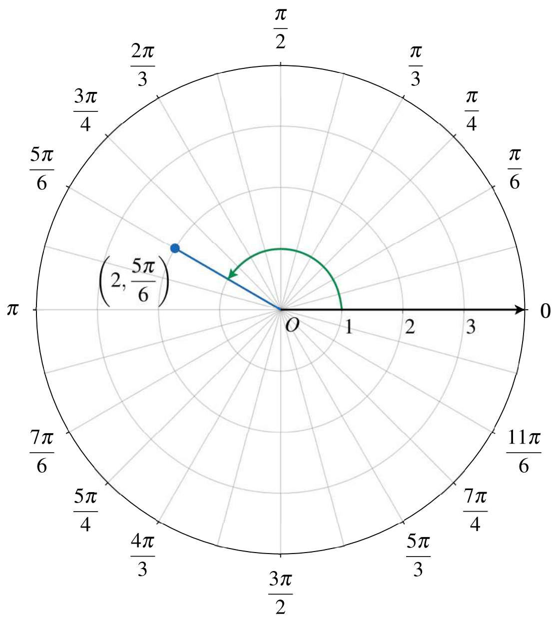
Thursday Night PreCalculus, March 7, 2024

Polar Coordinates

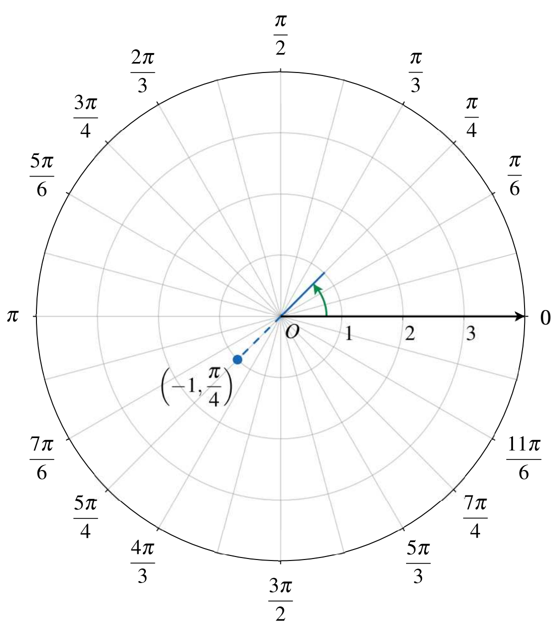
Problems

1. Plot the points whose polar coordinates are given.

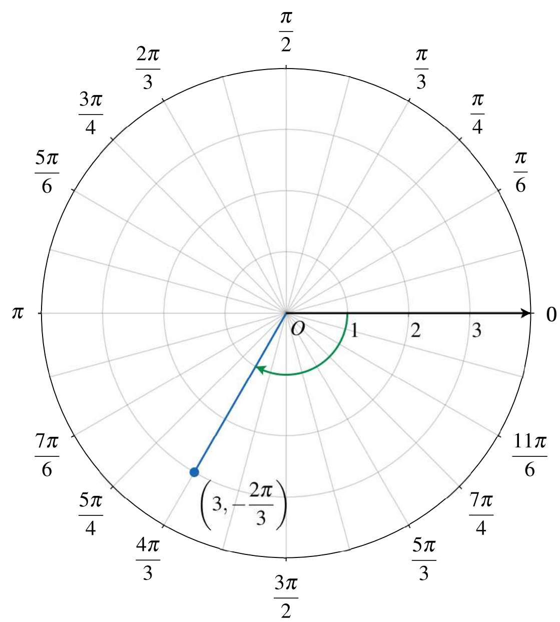
(a) $\left(2, \dfrac{5\pi}{6}\right)$



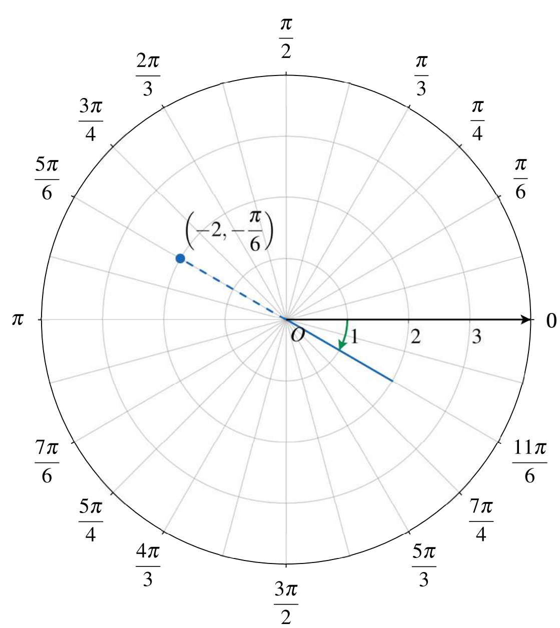
(b) $\left(-1, \dfrac{\pi}{4}\right)$



(c) $\left(3, -\dfrac{2\pi}{3}\right)$



(d) $\left(-2, -\dfrac{\pi}{6}\right)$



2. Convert the polar coordinates to rectangular coordinates.

(a) $\left(\sqrt{2}, \frac{5\pi}{3}\right)$

$$x = \sqrt{2} \cdot \cos\frac{5\pi}{3} = \sqrt{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2}$$

$$y = \sqrt{2} \cdot \sin\frac{5\pi}{3} = \sqrt{2} \cdot \left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{\frac{3}{2}}$$

The point is represented by $\left(\frac{\sqrt{2}}{2}, -\sqrt{\frac{3}{2}}\right)$ in Cartesian coordinates.

(b) $\left(-2, -\frac{\pi}{6}\right)$

$$x = -2 \cdot \cos\left(-\frac{\pi}{6}\right) = -2 \cdot \frac{\sqrt{3}}{2} = -\sqrt{3}$$

$$y = -2 \cdot \sin\left(-\frac{\pi}{6}\right) = -2 \cdot \left(-\frac{1}{2}\right) = 1$$

The point is represented by $(-\sqrt{3}, 1)$ in Cartesian coordinates.

3. Convert the rectangular coordinates to polar coordinates.

(a) $(2, 2\sqrt{3})$

$$r = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = \sqrt{16} = 4$$

$$\tan\theta = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

The point $(2, 2\sqrt{3})$ lies in the first quadrant. Choose $\theta = \dfrac{\pi}{3}$

The point is represented by $\left(4, \dfrac{\pi}{3}\right)$ in polar coordinates.

(b) $(-1, 2)$

$$r = \sqrt{(-1)^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5}$$

$$\tan\theta = \frac{2}{-1} = -2 \quad \Rightarrow \quad \theta = \tan^{-1}(-2), \quad \theta = \tan^{-1}(-2) + \pi$$

The point is represented by $(\sqrt{5}, \tan^{-1}(-2) + \pi)$ in polar coordinates.

4. Express the complex number $1 - i$ in the polar form $(r\cos\theta) + i(r\sin\theta)$.

The complex number $1 - i$ has rectangular coordinates $(1, -1)$.

$$r = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$\tan\theta = \frac{-1}{1} = -1 \quad \Rightarrow \quad \theta = -\frac{\pi}{4}$$

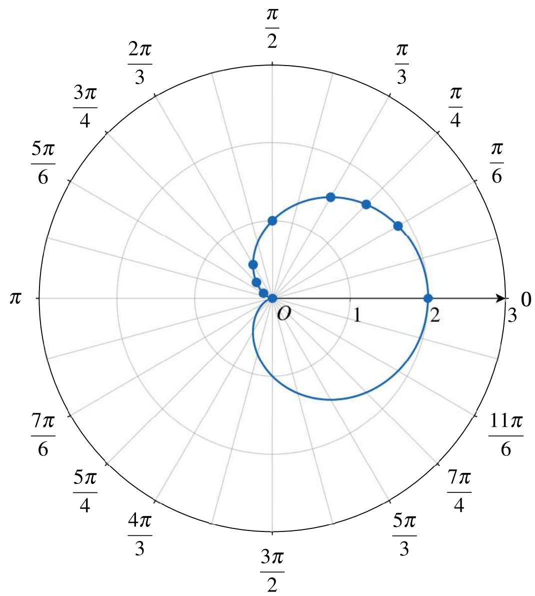
The complex number can be written as

$$\sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\left(\sqrt{2}\sin\left(-\frac{\pi}{4}\right)\right) = \sqrt{2}\cos\left(\frac{\pi}{4}\right) - i\left(\sqrt{2}\sin\left(\frac{\pi}{4}\right)\right)$$

5. Create a table of values to sketch each polar graph. Use technology to check your work.

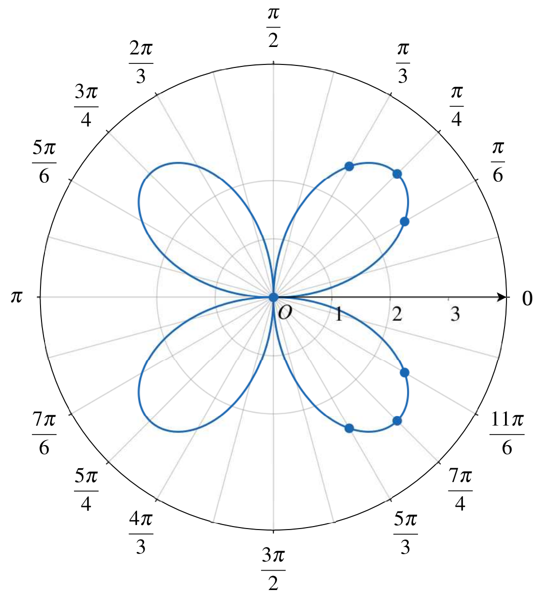
(a) $r = 1 + \cos\theta$

θ	$r(\theta)$
0	2
$\frac{\pi}{6}$	$1 + \frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$1 + \frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{3}{2}$
$\frac{\pi}{2}$	2
$\frac{2\pi}{3}$	1
$\frac{3\pi}{4}$	$1 - \frac{\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$1 - \frac{\sqrt{3}}{2}$
π	0



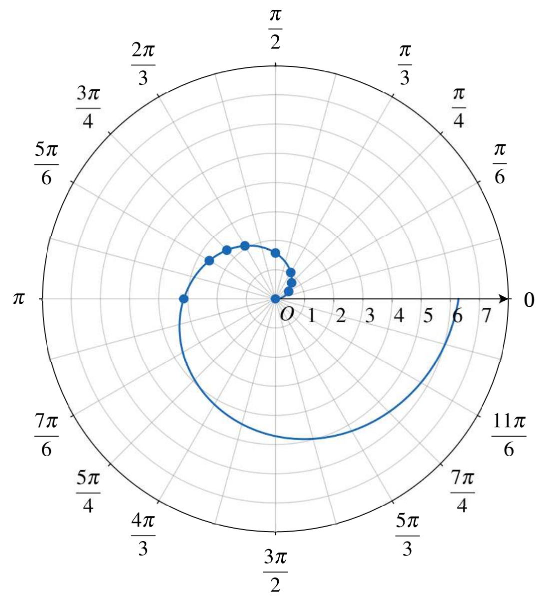
(b) $r = 3\sin(2\theta)$

θ	$r(\theta)$
0	0
$\frac{\pi}{6}$	$\frac{3\sqrt{3}}{2}$
$\frac{\pi}{4}$	3
$\frac{\pi}{3}$	$\frac{3\sqrt{3}}{2}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$-\frac{3\sqrt{3}}{2}$
$\frac{3\pi}{4}$	-3
$\frac{5\pi}{6}$	$-\frac{3\sqrt{3}}{2}$
π	0



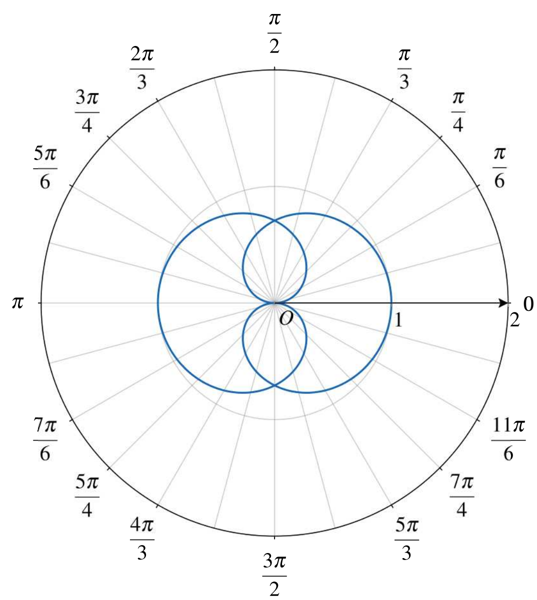
(c) $r = \theta, \quad \theta \geq 0$

θ	$r(\theta)$
0	0
$\frac{\pi}{6}$	$\frac{\pi}{6}$
$\frac{\pi}{4}$	$\frac{\pi}{4}$
$\frac{\pi}{3}$	$\frac{\pi}{3}$
$\frac{\pi}{2}$	$\frac{\pi}{2}$
$\frac{2\pi}{3}$	$\frac{2\pi}{3}$
$\frac{3\pi}{4}$	$\frac{3\pi}{4}$
$\frac{5\pi}{6}$	$\frac{5\pi}{6}$
π	π



6. Consider the polar function $r(\theta) = \cos\left(\dfrac{\theta}{2}\right)$ for $0 \le \theta \le 4\pi$.

(a) Graph the polar function over the given domain.



(b) Find the average rate of change of r with respect to θ over the interval $0 \le \theta \le \dfrac{\pi}{2}$. Is the radius increasing or decreasing over the given interval? Explain your reasoning.

$$\frac{r\left(\frac{\pi}{2}\right) - r(0)}{\frac{\pi}{2} - 0} = \frac{\cos\frac{\pi}{4} - \cos 0}{\frac{\pi}{2}} = \frac{\frac{\sqrt{2}}{2} - 1}{\frac{\pi}{2}} = \frac{2}{\pi} \cdot \left(\frac{\sqrt{2}}{2} - 1\right)$$

The radius is decreasing over the interval $0 \le \theta \le \dfrac{\pi}{2}$.

As θ increases in this interval, $\dfrac{\theta}{2}$ also increases, and $\cos\left(\dfrac{\theta}{2}\right)$ decreases.