

# Monopoly - Part 1

## Teacher Notes & Answers

7 8 9 10 11 12



TI-Nspire™



Activity



Student



90 min

## Introduction

Monopoly™ is one of the most popular board games in the world. The game is so successful it has become an eponym for the definition: “exclusive possession or control”, evident via a simple internet search on the word *monopoly*. Most people have their favourite property set to collect; most commonly the blue property set adjacent to GO. At \$350 and \$400 they are the most expensive on the board. Are these properties really the best?

If you're going to be a property tycoon, you should do your research before investing. first. This three-part investigation looks at different ways to explore which properties on the Monopoly board represent the best value for money and arm you with mathematically based strategies before you roll the dice in your next game.

### Teacher Notes:

Some students may not have played Monopoly before, while it is not critical, it is highly desirable and will make the learning experience much more memorable. Student familiarity with the game adds to the value of the context for this three-part investigative series and the follow up investigation on actual Real-Estate prices and trends. The names associated with each square vary, depending on which version students own/play. For example, in the USA, the first two properties are titled: Mediterranean and Baltic Avenues, the UK and AU versions had Old Kent and Whitechapel Roads. In the revised Australian edition these properties are called Todd and Smith streets.

Property references in this activity cite the original names from the AUS / UK version but do not alter the mathematical content.

The investigation aims to help students understand gradient, y-intercept, lines of ‘best fit’ and linear regression.



## Equipment

- Monopoly board game
- TI-Nspire Calculator

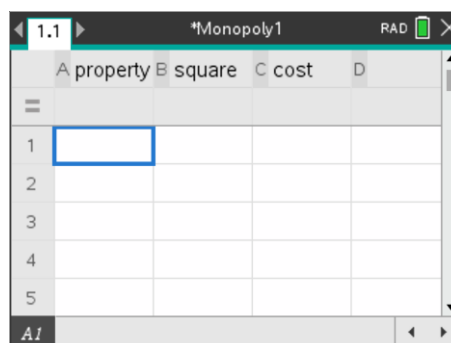
## Instructions

Start a new TI-Nspire document and insert a spreadsheet.

Create the following list names:

- Property
- Square
- Cost

Save the file as: Monopoly1



In this investigation ONLY property prices will be recorded, utilities and railroads will not be included. The “GO” square represents 0, so the first property appears on the first space, the second property is on the third and so on.

**Record the name, location (square) and cost** for all the available properties on the board. The first side of the board has been completed in the spreadsheet shown opposite.

**Note:** Property names may vary depending on the game version.

	A property	B square	C cost	D
1	Old Ken...	1	60	
2	Whitech...	3	60	
3	The Ang...	6	100	
4	Euston R...	8	100	
5	Pentonv...	9	120	

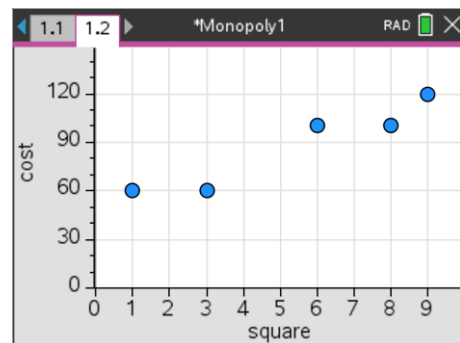
### Question: 1.

Describe the general trend for property prices as a player progresses around the board.

**The further a player moves around the board the higher the property price.**

Insert a Data & Statistics application and produce a scatter plot with **square** on the independent (horizontal) axis and **cost** on the dependent (vertical) axis.

Data for the first side of the board is shown opposite.



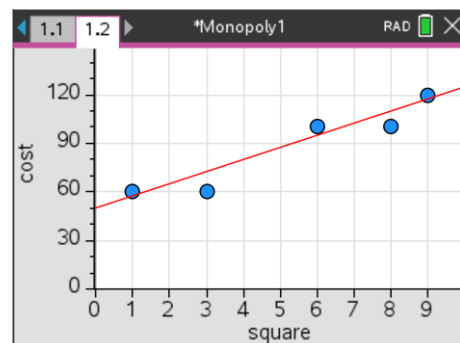
By default the graph zooms in on the data being displayed. For this data, zero will not be displayed on the *square* or *cost* axes. Use the Window - Zoom option to adjust x-min and y-min to include zero. Including the zero on both axes helps identify the relative location of the data points and therefore understand the relationship.

It is not possible for a single straight line to pass through all the data points, however you can imagine a straight line passing as close as possible to data points, this is referred to as a “line of best fit”.

Determine, by approximation only, the equation to a line that would best represent or fit the data. This equation can be changed manually by experimentation.

To graph your equation press: **Menu > Analyse > Plot Function**

A line representing the first couple of data points has been shown opposite; the equation itself has been hidden.



### Question: 2.

Explain how you determined your original equation, the subsequent adjustments you made and record your final equation.

ie: What estimates were made to help determine the gradient and y intercept of your equation(s)?

**Answers will vary: Typical strategies include using the price of Mayfair (or its equivalent) as \$400 being 40 spaces from Go and subsequently estimating the gradient as 10. This approach assumes prices are directly proportional**

to spaces. The data clearly shows that GO is not equivalent to a property value of \$0.00, so the final gradient estimates should be less than 10.

- Using the first and last property values with their respective locations produces a gradient of approximately 8.7
- Averaging the first property set price and location and applying a similar strategy to the last property set produces a gradient of approximately 8.5
- Both of the above strategies assume that Mayfair and Parklane form a reasonable fit, applying the above strategies to other property groups produces a gradient much closer to a value of 8.
- Using the gradient as a value between 8 and 10 means the y – intercept should fall within the range: \$30 to \$52. These values can be achieved by using reasonable gradient values (as described above) and extrapolating back to 'Go' (y – intercept) from combinations of the first three spaces on the board.  
A final equation of best fit, estimated by students should appear close to:  $y = 8.5x + 43$



By default the plotted function is recorded in  $f_1(x)$ . This function can be called upon in any application within this problem. Example in the calculator application type:  $f_1(11)$  to estimate the value of the first property on the second side of the board.

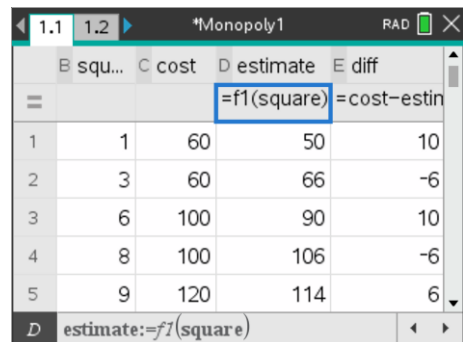
**Question: 3.**

Reproduce the table below for **all sides** of the Monopoly board. Record the actual and estimated property values (estimated from your rule). [Difference = cost - estimate]

Spaces	1	3	6	8	9	11	13	14
Actual	60	60	100	100	110	140	140	160
Estimate								
Difference								

Answers will vary depending on the student's equation. Students should consider using technology, as shown opposite to determine the result, this will also make Question 4 easier!

The example shown uses:  $f_1(x) = 8x + 42$



**Question: 4.**

Calculate and record the 'mean' difference. Explain why this calculation is not a true representation of the accuracy of your line.

Answers will vary depending on the student's equation. Students can use the calculator application to determine the mean (average) difference.

A quick look through the difference list should reveal most differences are higher than the average (6.27). The problem is that the differences include a mixture of positive and negative values, therefore the mean of the absolute value provides a better estimate of how well the line predicts property values.

Expression	Result
$\text{mean}(\text{diff})$	$\frac{69}{11}$
$\text{mean}(\text{diff})$	6.27273
$\text{mean}( \text{diff} )$	9

### Question: 5.

Square all the differences, compute the *mean* of the squared values and then *square-root* the result.

- (i) Record this new computed value.

Answers will vary, the value can be computed using:  $\text{mean}(\text{difference}^2)$  from the calculator application when the differences have been computed in a list and labelled accordingly.

Typically this value will be of the order 150+, after square-rooting the result, the average difference is approximately: 12.4



- (ii) Explain the effect that 'squaring' has on the differences.

Squaring removes the negatives, but the process also makes the numbers much larger.

### Teacher Notes:

The purpose of this line of questioning is to build towards 'residual squares'. Squaring the differences removes the negative sign from the computed difference values and may be represented visually as a square. These squares can be shown on TI-Nspire alongside the regression equation!

- (iii) Explain why the 'square-root' step is included.

The square-root is designed to 'undo' the squaring operation.

### Question: 6.

Return to the Statistics application. From the **Analyse** menu, select **Residuals** followed by **Show Residual Plot**. Move the cursor over the data points on this plot. What does this plot represent?



Click on a point in the residual plot to see the corresponding point in the scatter plot.

The residual plot shows how far the actual data point is compared with the value determined by the defined rule.

### Question: 7.

From the **Analyse** menu select **Regression** followed by **Linear Regression**. Explain what this line does and compare it to your line including specific reference to the 'residuals'.

The regression line is a statistical measure of the 'line of best fit' by minimising the residuals. (Minimises the sum of the squared residuals).

**Question: 8.**

Does the relationship between the property price and distance from 'GO' provide any information about which property represents the best value?

No.

**Teacher Notes:**

The purpose of this question is to highlight that just because a correlation exists, doesn't necessarily give it meaning. In contrast, actual property prices generally decrease as you move further away from the CBD (Central Business District). An investigation into this is provided in the "Real Estate" investigation.

**Question: 9.**

Explain why the property prices (and perceived value) might increase as a player progresses around the board.

If the most 'valued' properties were located on the first side of the board, there would be a distinct advantage to going first. The first player is guaranteed to land on one of the first 12 squares before their opponent; however, there is a chance (albeit smaller) that the second player might reach the fourth side of the board first. Furthermore, if the most 'valued' properties are located on the last side of the board, players must consider their first purchases, over-spending early may lead to a lack of funds to purchase the more expensive properties.

**Teacher Notes:**

Students are not required to determine exact (or experimental) probabilities for this question, although it could be included as a challenging problem to solve: "What is the likelihood that the second player will reach the fourth side of the Monopoly board first?"

As covered in the 'answer' (above), students should be able to reconcile a probability of 'zero' (player two lands on one of the first 12 squares first) with the probability (albeit smaller) that they land on the last side of the board first, therefore an increased probability making the game a little fairer.